the GEODESIC

Ensemble Newsletter
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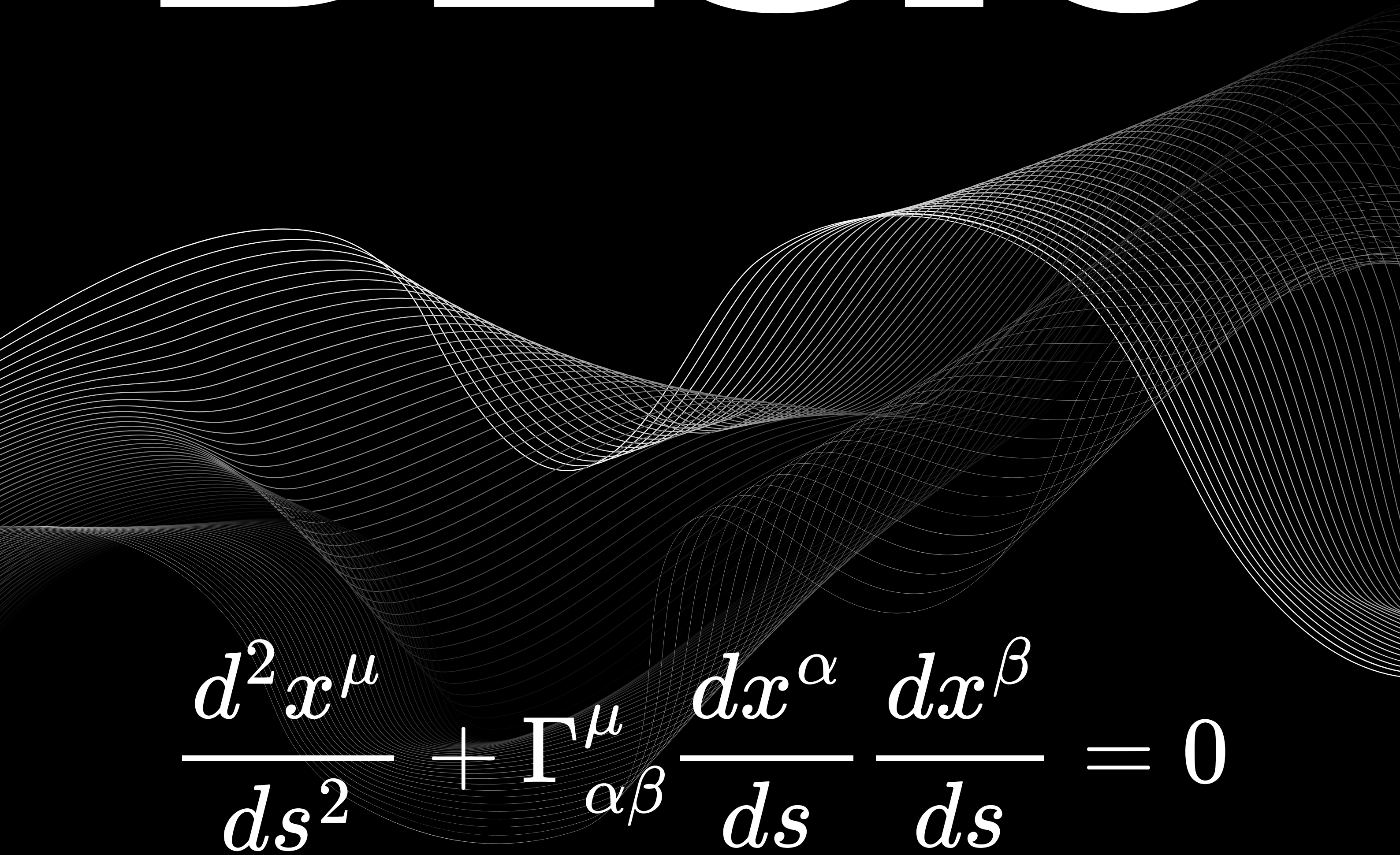

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0$$

FROM THE EDITORS

Welcome to the first issue of **The Geodesic**!

The Geodesic is an attempt to bring the frontiers of physics closer to curious minds. We aim to cater to a wide audience, whether you are a beginner eager to explore fascinating physics concepts, or someone looking to dive deeper into the mathematical details behind them.

In this issue, we dive into some of the most exciting topics in physics and feature conversations with leading researchers regarding their experiences in research, teaching and collaboration with undergraduate students. From cutting-edge theoretical developments like the Newman–Janis algorithm and Emergent Gravity to fascinating experimental breakthroughs in Graphene and 2.5D materials, our articles span the spectrum of modern physics.

As you turn these pages, we invite you to embark on a journey through prose that captures the spirit of physics in words and articles that reveal its beauty through mathematical precision, from the fundamentals that ground our knowledge to the discoveries that push its boundaries.

Whether you're a student, a researcher, or an avid enthusiast, we hope these pages inform, inspire, and ignite your passion for understanding the universe. Our goal is to make physics approachable, engaging, and thought-provoking, encouraging you to ask questions, explore ideas, and see the world through the lens of scientific curiosity.

CONTENTS

THE NEWMAN-JANIS ALGORITHM: MATHEMATICAL PHYSICS AT ITS INGENIOUS BEST

Sriraj Chandra

The Schwarzschild metric: The Schwarzschild metric, named after Karl Schwarzschild, was the first exact nontrivial solution to Einstein's field equations to be found. It is also the simplest, by far, of the different metrics we will encounter in the article. It describes the gravitational field outside a spherically symmetric mass, with zero charge, zero angular momentum, in the case of a zero universal cosmological constant. Despite its simplicity, it describes two singularities,one physical and one coordinate(nonphysical), which later inspired the theoretical framework of black holes. One of the rather popular terms the metric has produced is the Schwarzschild radius, defined as $\frac{2GM}{c^2}$. A body with a radius smaller than its Schwarzschild radius inevitably collapses to form the famed ``black hole'', a term coined by John Archibald Wheeler, one of the pioneers of general relativity. This appears as the coordinate singularity which later inspired the theoretical framework of black holes, Schwarzschild metric and is also the radius for the event horizon, a popular Newtonian derivation, equating the escape velocity of a spherically symmetric mass to the speed of light, reproduces the same expression, which provides the ``black'' i.e. light-devoid attribute to this physical object. Let us now present the metric, in Schwarzschild own coordinates:

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

(Here, we work in the God given units i.e. c = G = h =1.) Is this the only possible solution to Einstein's field equations with the given conditions? It turns out that it is. Birkhoff's theorem, proven by David Birkhoff, states exactly this, i.e. any spherically symmetric solution of the vacuum field equations must also be stationary and asymptotically flat, which are the same assumptions one takes in order to derive the Schwarzschild metric. Thus, the Schwarzschild metric universally describes the gravity outside a spherically symmetric non-charged non-rotating body. However, one can see that the metric is limited in its applicability, i.e. very few cosmological objects can actually be described by this metric. Let us now move to a much more exciting albeit complicated solution of the field equations - the Kerr metric.

The Kerr metric: Derived by Roy Kerr in 1963, a long 47 years after the Schwarzschild metric was derived for the first time, it is an exact solution of the vacuum Einstein field equations, which describes the gravity outside a non-charged rotating spherically symmetric mass. There are multiple interesting consequences of this metric, the most studied of which is the Lense-Thirring effect. Roughly speaking, this effect predicts that objects coming close to a rotating mass will be entrained to participate in its rotation, not because of any applied force or torque that can be felt, but rather because of the swirling curvature of spacetime itself associated with rotating bodies. The ergosphere is defined as the region of a black hole outside its event horizon before the static limit. A quaint use of the ergosphere is described by the Penrose process, theorised by Sir Roger Penrose in 1969. In this process, matter falling into the ergosphere comes out with more energy than it entered with as long as a part of it goes into the singularity. Thus, the body is able to extract the rotational energy of the rotating black hole, and, as an effect, slows it down. The Kerr metric, in Boyer-Lindquist coordinates, is given by:

$$g_{\mu\nu} = \begin{pmatrix} 1 - \frac{2Mr}{\Sigma} & 0 & 0 & \frac{2Mar\sin^2\theta}{\Sigma} \\ 0 & -\frac{\Sigma}{\Delta} & 0 & 0 \\ 0 & 0 & -\Sigma & 0 \\ \frac{2Mar\sin^2\theta}{\Sigma} & 0 & 0 & -\left(r^2 + a^2 + \frac{2Ma^2 r\sin^2\theta}{\Sigma}\right)\sin^2\theta \end{pmatrix}$$

where, $\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2Mr + a^2$, a is the spin parameter defined as $a = \frac{J}{M}$, J is the angular momentum and M is the mass.

We get the line element form of the metric just like the one we have for the Schwarzschild metric from the equation $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$.

The time gap between these two solutions can be attributed to the difference in their complexities. The Schwarzschild solution can be derived in a straightforward process just using basic analysis and is covered in almost all courses on general relativity worldwide. On the other hand, the derivation of the Kerr solution requires detailed knowledge of algebraically special metrics and deals with solving the Cartan structure equations. It is orders higher in complexity.

The Algorithm : This is where the Newman-Janis algorithm proves to be extremely useful. After Roy Kerr gave the Kerr solution in 1963, Ezra Newman and Allen Janis struck ``mathematical'' gold in 1964. In a paper modestly titled ``Note on the Kerr Spinning Metric'', they demonstrated their findings of a coordinate complexification and transformation technique that directly gave the Kerr solution from the Schwarzschild solution, bridging 47 years of work in a 2 page paper. In this article, we will go through the same technique and show how useful it is, evident by the fact that the solution for gravity outside a rotating charged spherically symmetric mass (Kerr-Newman metric) was derived from the solution for gravity outside a non-rotating charged spherically symmetric mass (Reissner-Nordstrom metric), using the same technique. We'll describe these two metrics after our exposition of the algorithm.

Let us go through the algorithm step-by-step, as described in the 1964 paper.

Step 1: Standard to Eddington-Finkelstein coordinates :

$$ds^2 = (1-\frac{2M}{r})dt^2 - (1-\frac{2M}{r})^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2)$$

This is the Schwarzschild line element in standard coordinates where c and G are both equal to 1. We now implement the following coordinate transformation.

$$\left(u, r', \theta', \phi'\right) = (t - r - 2Mln(r-2M), r, \theta, \phi)$$

For convenience, we drop the prime notation. In the new coordinates, the line element takes the form:

$$ds^2 = (1-2M/r)du^2 + 2dudr - r^2(d\theta^2 + sin^2\theta d\phi^2)$$

The contravariant metric tensor in this coordinates is given by:

$$g^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & \frac{2M}{r}-1 & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\sin^2\theta} \end{pmatrix}$$

Step 2: Expression as a null tetrad : In the last step, the metric we derived can be written as :

$$g^{\mu\nu} = l^\mu n^\nu + l^\nu n^\mu - m^\mu \bar{m}^\nu - m^\nu \bar{m}^\mu$$

where,

$$l^\mu = \delta_1^\mu, n^\mu = \delta_0^\mu - \frac{1}{2}(1-\frac{2M}{r})\delta_1^\mu,$$

$$m^\mu = \frac{1}{\sqrt{2}r}(\delta_2^\mu + \frac{i}{sin\theta}\delta_3^\mu), \bar{m}^\mu = \frac{1}{\sqrt{2}r}(\delta_2^\mu - \frac{i}{sin\theta}\delta_3^\mu)$$

Step 3: Complexification : The radial coordinate is complexified i.e., the tetrad is rewritten as:

$$l^\mu = \delta_1^\mu, n^\mu = \delta_0^\mu - \frac{1}{2}(1 - M(\frac{1}{r} + \frac{1}{\bar{r}}))\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}r}(\delta_2^\mu + \frac{i}{sin\theta}\delta_3^\mu)$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}\bar{r}}(\delta_2^\mu - \frac{i}{sin\theta}\delta_3^\mu)$$

Step 4: Complex coordinate transformation : The following coordinate transformation is carried out :

$$\left(u', r', \theta', \phi'\right) = (u - iacos\theta, r + iacos\theta, \theta, \phi)$$

on l_μ, n_μ and m_μ .($\bar{m}_\mu$ is defined as the complex conjugate of m^μ.) r' and u' are allowed to be real. This results in the following tetrad.

$$(l')^\mu = \delta_1^\mu, (n')^\mu = \delta_0^\mu - \frac{1}{2}(1 - 2M\frac{r'}{r'^2 + a^2cos^2\theta})\delta_1^\mu$$

$$(m')^\mu = \frac{1}{\sqrt{2}(r' + iacos\theta)}(iasin\theta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{i}{sin\theta}\delta_3^\mu)$$

$\bar{m}_\mu$ is defined as the complex conjugate of m_μ . Consequently, we obtain the metric:

$$(g')^{\mu\nu} = (l')^\mu (n')^\nu + (l')^\nu (n')^\mu - (m')^\mu (\bar{m}')^\nu - (m')^\nu (\bar{m}')^\mu$$

which has the following non-zero contravariant components:

$$g^{\mu\nu} = \begin{pmatrix} -\frac{a^2\sin^2\theta}{\Sigma} & \frac{r^2+a^2}{\Sigma} & 0 & -\frac{a}{\Sigma} \\ \frac{r^2+a^2}{\Sigma} & -\frac{\Delta}{\Sigma} & 0 & 0 \\ 0 & 0 & -\frac{1}{\Sigma} & 0 \\ -\frac{a}{\Sigma} & 0 & 0 & -\frac{1}{\Sigma\sin^2\theta} \end{pmatrix}$$

Step 5: Eddington-Finkelstein to Boyer-Lindquist coordinates: The required transformation is given, from (u, r, θ, ϕ) (denoted as z-coordinates) to (t, r′, θ′, ϕ′)(denoted as x-coordinates).

$$\left(dt, dr', d\phi', d\theta'\right) = \left(du + \frac{a^2+r^2}{\Delta}dr, dr, d\phi + \frac{a}{\Delta}dr, d\theta\right)$$

Again, we drop the prime notation. Now, we transform the contravariant form of the metric ((2,0) tensor) using the relation:

$$(g')^{\mu\nu} = g^{\alpha\beta}\frac{\delta z^\mu}{\delta x^\alpha}\frac{\delta z^\nu}{\delta x^\beta}$$

Then the resulting metric is inverted to obtain the covariant form, which can then be compared with the form of the Kerr metric given in the last section. The readers are encouraged to carry this out themselves and check if this holds true. (Hint: If not, a change of signature might be needed!)

This completes the algorithm in its entirety. We discuss some modifications next.

The Reissner-Nordstrom and Kerr-Newman metrics: A slightly modified algorithm:

The Reissner-Nordstrom metric is the immediate successor to the Schwarzschild metric, having been independently derived by multiple physicists from 1916 to 1921. It is the metric due to a spherically symmetric non-rotating charged mass of charge Q, given by the line element:

$$ds^2 = f(r)dt^2 - (f(r))^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

where, $f(r) = \left(1 - \frac{2M}{r} + \frac{r_Q^2}{r^2}\right)$, $r_Q = \frac{Q^2}{4\pi\epsilon_0}$ and $G = c = 1$.

Despite being known since the 1910s, the Kerr-Newman metric, its rotating counterpart, was not derived until 1965, the very same year the Newman-Janis algorithm was discovered. This is not a mere coincidence, as, in fact, the very first derivation of the Kerr-Newman metric was done by applying the algorithm to the Reissner-Nordstrom metric, in the paper titled ``Metric of a Rotating, Charged Mass'' by E. T. Newman, E. Couch, K. Chinnapared, A. Exton, A. Prakash, and R. Torrence. In this one and a half page paper, the authors just provide the steps of the transformation almost similar to the aforementioned algorithm, with very slight changes to suit the metric. The Kerr-Newman metric, in Boyer-Lindquist coordinates, is given by the metric $g_{\mu\nu} =$

$$\begin{pmatrix} \left(1 - \frac{2Mr - Q^2}{\Sigma}\right) & 0 & 0 & \frac{a(2Mr - Q^2)\sin^2\theta}{\Sigma} \\ 0 & -\frac{\Sigma}{\Delta} & 0 & 0 \\ 0 & 0 & -\Sigma & 0 \\ \frac{a(2Mr - Q^2)\sin^2\theta}{\Sigma} & 0 & 0 & -\left(r^2 + a^2 + \frac{(2Mr - Q^2)a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta \end{pmatrix}$$

where:

$$\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2Mr + a^2 + Q^2$$

This was a demonstration of how phenomenal the algorithm is. It is used even today to convert non-rotating solutions to rotating solutions of the field equations in different f(R) theories of gravity. However, as physicists, we also strive to understand the meaning of this seemingly arbitrary transformation. There have been numerous papers beginning from the original by Newman and Janis to that very object, and have led to multiple physical interpretations of this technique.

Interpretation: The original paper does not give an interpretation of the algorithm. However, the authors state that in a private communication, R. Kerr had shown that the algorithm works for a special class of solutions, given by the form

$$g_{\mu\nu} = \eta_{\mu\nu} + \lambda^2 l_\mu l_\nu$$

where $\eta_{\mu\nu}$ is the Minkowski metric and l_μ denotes the same entity as used in the algorithm.

The Schwarzschild metric happens to satisfy that criteria. However, that need has been met amply by a multitude of papers on that very subject. Still, there is a lack of a clear explanation. At one extreme, there lies the view that the NJA is a “fluke” that merely tricks one’s eyes. In the middle ground, there lies most of works viewing the NJA as a noteworthy technical trick. This line of research focuses on resolving some of the algorithm’s inherent ambiguities, identifying underlying assumptions, or testing its generalizations to larger classes of solutions. Finally, the other extreme arises from the serious stance taken by Newman himself. For him, the algorithm must be signaling a deeper physical principle, which he speculates as an electric-magnetic duality between orbital and spin angular momenta. We discuss one such interpretation.

From the paper titled, "An Explanation of the Newman-Janis Algorithm", by S.P. Drake and P. Szekeres: The authors prove the uniqueness of certain solutions generated by the NJA. The results are stated as follows:-

1. The only perfect fluid generated by the Newman-Janis Algorithm is the vacuum i.e. the Kerr metric.
2. The only algebraically special spacetimes generated by the Newman Janis algorithm are Petrov type D.
3. The only Petrov type D spacetime generated by the Newman Janis algorithm with a vanishing Ricci scalar is the Kerr-Newman spacetime.

The meaning of the keywords are given hereby: ``Algebraically special'' and ``Petrov type D''

To understand these terms, we must have an idea about the Petrov classification. Although we do not delve into the mathematics of it, the process of classification is based on the symmetries of the Weyl tensor. There are 5 Weyl scalars $\Psi_j, (j = 0, 1, 2, 3, 4)$ which distinguish the six different Petrov types. Each of these types describe different gravitational spacetimes. If $\Psi_0 = \Psi_1 = 0$, the spacetime is called algebraically special. The Kerr metric is classified as Petrov type D, i.e. $\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0.$ Generally, Petrov type D spacetimes occur as the exterior field of a gravitating object which is completely characterized by its mass and angular momentum. The authors also discuss the ambiguity of complexification of coordinates in the NJA. The conclusion is drawnthat the particular choice of complexification used in the standard NJA to generate the Kerr-Newman solution are not arbitrary, but could in fact be chosen in no other way in order for the NJA to be successful at all. This provides, in a sense, an “explanation” of the algorithm.

Scope of the algorithm:

The algorithm, in its different generalized forms, has been used extensively in metric transformations. We list a few of them, other than the ones we discussed earlier.

- f(R) gravity theories
- Einstein–Maxwell–Dilaton theories

However, it doesn't return expected results on Braneworld and Born–Infield theories.

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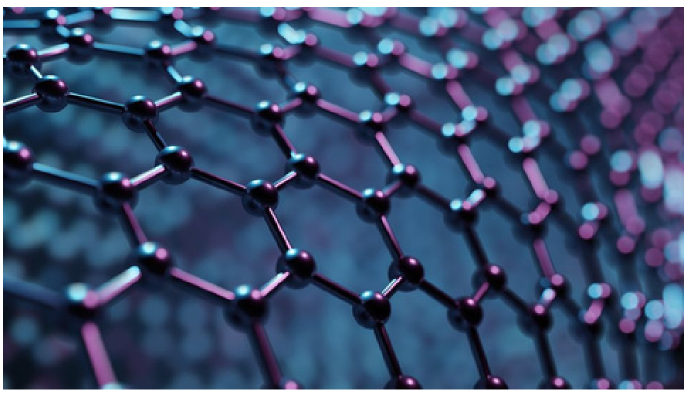
BEYOND FLATLAND: THE RISE OF 2.5D MATERIALS IN NEXT-GENERATION TECHNOLOGY

Amrita Notani

Introduction: In 2004, a simple experiment with Scotch tape and graphite sparked a scientific revolution. The isolation of graphene, which is a one-atom-thick sheet of carbon, revealed that materials can exist in just two dimensions and possess extraordinary properties. This discovery challenged our understanding of condensed matter physics and opened up possibilities for electronics beyond silicon. However, despite their impressive potential, these purely two-dimensional (2D) materials have significant limitations. They exhibit poor light absorption, are mechanically fragile, and have limited diversity in their properties.

This brings us to the world of 2.5D materials. This emerging area occupies the space between 2D and 3D. Unlike strictly planar materials, these engineered systems blend the quantum benefits of atomically thin materials with the practical strength of three-dimensional structures. Picture layered designs where each layer has a specific function, while the entire structure operates in ways that a single layer cannot. This captures the essence of 2.5D materials, a balanced approach that maintains quantum confinement while allowing for greater functionality.

The timing of this development is critical. As traditional semiconductor technology nears its physical limits, 2.5D materials present a path forward for computing, energy conversion, and sensing technologies. They signify not just a small improvement, but a real shift in how we think about and create materials at the nanoscale.



This review outlines the rise of this exciting field. It covers how scientists are building these dimensional hybrids, the unique physical phenomena that occur at their interfaces, and the technological advancements they may lead to. From quantum computing to energy storage, 2.5D materials have the potential to transform our technological landscape, provided we can understand and manage their complexities.

Classification: Building the Third Half-Dimension:

To understand 2.5D materials, think of them as architectural variations on a theme. They start with atomically thin building blocks, but are assembled in ways that create something greater than the sum of their parts. This section presents a taxonomy of these structures, moving from simple to complex.

Van der Waals heterostructures represent the most straightforward approach to 2.5D construction. These are vertical stacks of different 2D materials held together by weak forces between layers. These molecular "layer cakes" maintain the integrity of each part while enabling new behaviors. For instance, when graphene is layered with hexagonal boron nitride (h-BN), the h-BN acts as an insulating base that significantly improves graphene's electronic performance by shielding it from environmental disturbances. Adding a third layer of a semiconductor like MoS_2 can create an ultra-thin transistor with performance that beats conventional silicon.

One of the most intriguing cases occurs when two identical graphene layers are slightly misaligned in rotation, as seen in twisted bilayer graphene. At specific "magic angles" around 1.1°, this simple adjustment transforms ordinary graphene into a superconducting material that can conduct electricity with zero resistance. This finding in 2018 sent waves through the physics community, showing how dimensional engineering can reveal unique properties not possible with traditional materials.

While vertical stacking builds upward, **lateral heterostructures** spread horizontally. They connect different 2D materials edge-to-edge within a single plane. Picture a patchwork quilt where each section has unique electronic properties, yet they form a seamless whole. These structures are especially promising for making electronic circuits at the atomic scale, where various functional regions like semiconductors, conductors, and insulators can be incorporated into a single atomically thin sheet.

Intercalated structures take a different approach by inserting foreign atoms or molecules between layers of a host material. This is similar to placing spacers between books on a shelf, changing the spacing and interactions among adjacent layers. For example, inserting lithium ions between layers of graphite creates lithium-intercalated graphite, which is now the standard anode in lithium-ion batteries. It hosts lithium ions between its carbon layers to store energy. In a similar vein, calcium-intercalated graphene shows potential as a high-temperature superconductor, and copper ions between MoS2 layers can improve catalytic activity for hydrogen production.

The most complex 2.5D materials are hybrid multidimensional architectures that integrate components of different dimensionalities. These might combine 2D sheets with 0D quantum dots (tiny semiconductor particles that exhibit size-dependent properties), 1D nanotubes (rolled sheets that form molecular wires), or even 3D nanostructures. An example includes MXene sheets decorated with platinum nanoparticles, which make highly efficient catalysts for fuel cells by maximizing the active surface area while keeping electrical conductivity through the 2D framework.

Finally, functionalized and reconstructed networks change flat 2D sheets into 3D architectures through controlled deformation—folding, wrinkling, or chemical modification. These structures often have improved mechanical properties and greater surface area. Crumpled graphene oxide, for example, forms springy particles that can be compressed multiple times without permanent deformation, making them useful for energy storage and mechanical dampening.

This diverse group of 2.5D architectures shows how controlling matter across traditional dimensional classes opens new paths for materials design that were not accessible before.

Physical Principles: When Dimensions Collide:

Physical Principles: When Dimensions Collide

The unique behavior of 2.5D materials comes from a key tension: the quantum physics of atomically thin layers interacts with the classical world of large-scale actions. This intersection forms a rich area of physical phenomena that can be used in technology.

At the core of these systems is quantum confinement with controlled coupling. In isolated 2D materials, electrons are limited to a plane, forming quantum wells that significantly change electronic behavior. In 2.5D systems, these quantum-confined layers interact in specific ways that engineers design. The strength of interlayer coupling becomes an adjustable factor, influenced by layer spacing, twist angle, and chemical makeup. This adjustability is best shown in twisted bilayer graphene. When two graphene sheets stack with a precise twist angle of 1.1°, their electronic bands interfere to create “flat bands”—energy states where electrons move so slowly they seem to be at rest. This increase in electron-electron interactions can lead to superconductivity and other unusual quantum phases. The twist angle acts like a “quantum knob” that can be tuned to access different physical effects, similar to turning a radio to find different stations.

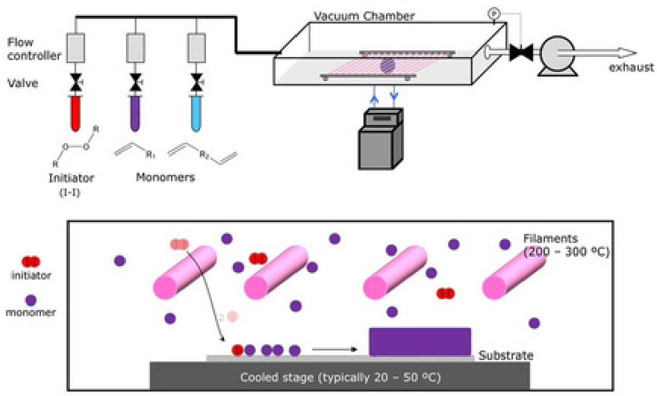
The layered structure of 2.5D materials leads to notable direction-dependent movement of electrons, photons, and heat, known as anisotropic transport. This anisotropy can be used for directing the flow of energy and information. For instance, in van der Waals heterostructures of graphene and hexagonal boron nitride, electrons travel freely along the graphene planes but face resistance when crossing between layers. This setup creates an “electron highway with speed bumps,” allowing designers to create transistors where current flows parallel to layers but can be controlled by a perpendicular electric field.

In addition, **moiré patterns** appear when similar crystal lattices overlap with slight misalignment. These patterns form a superlattice—a periodic potential that changes how electrons, phonons, and photons behave. Moiré engineering essentially turns 2D sheets into entirely new materials with properties that can be engineered by adjusting the twist angle. This work has led to the discovery of “moiré excitons” (bound electron-hole pairs) in twisted semiconductor bilayers, which show unique optical features and could lead to new quantum light sources.

The limited screening in these quasi-2D systems boosts **excitonic effects**. Excitons—pairs of bound electrons and holes—dominate the optical behavior of many 2D semiconductors because of lower dielectric screening. In 2.5D hetero-structures, excitons can form across different layers, creating “inter-layer excitons” where the electron is in one material and the hole in another. These separated particles have much longer lifetimes compared to conventional excitons, existing for nanoseconds instead of picoseconds. This greater durability opens up new possibilities in information processing since excitons can travel significant distances before recombining. Lastly, the controlled spacing and interaction between layers create a perfect setup for studying many-body physics—the collective interactions of large numbers of particles. In 2.5D materials, these interactions can produce new phenomena like superconductivity, magnetism, and topological phases that don't exist in the individual layers. The ability to finely tune these interactions by controlling layer composition and shape makes 2.5D materials outstanding platforms for exploring the edges of condensed matter physics.

Fabrication: The Art of Atomic Architecture:

Creating 2.5D materials requires precision at the atomic scale, similar to building a skyscraper where each floor must be positioned perfectly down to the last millimeter. This section explores key techniques that allow for nanoscale architectural control.



Chemical Vapor Deposition (CVD)

Chemical vapor deposition (CVD) has become the main method for growing high-quality 2D layers and heterostructures at scale. In this process, precursor gases react on a heated substrate surface, depositing atomically thin films with a controlled composition. The strength of CVD lies in its versatility. By adjusting parameters such as temperature profiles, gas flow, and substrate preparation, researchers can grow single-layer graphene, transition metal dichalcogenides, or complex heterostructures. For example, sequential CVD processes have created vertical stacks of graphene/h-BN/MoS2 with atomically clean interfaces, which is critical for electronic applications. This technique has advanced to allow wafer-scale production of some 2.5D systems, an important step for industrial viability. However, controlling layer orientation and preventing unintentional defects remains a challenge in CVD methods.

For research purposes, **mechanical exfoliation** and **deterministic stacking** provide unmatched precision for creating model 2.5D structures. This approach starts with the well-known "Scotch tape method," where atomically thin flakes are peeled from bulk crystals. Selected flakes are then transferred with micrometer precision using specialized equipment similar to microscopic "stamp pads." Using polymer carriers and alignment marks, researchers can determine where to place each layer and their rotational alignment with sub-degree accuracy.

This technique made possible the discovery of superconductivity in magic-angle twisted bilayer graphene, requiring twist-angle precision of about 0.1°. While it cannot be scaled for mass production, it is still key for testing new 2.5D designs and exploring fundamental physics.

Solution-based methods offer a compromise between precision and scalability. Techniques like liquid-phase exfoliation create suspensions of 2D materials that can be processed like inks. Layer-by-layer deposition—through spin-coating, spray coating, or inkjet printing—then builds up complex structures in a controlled way. These approaches are especially useful for applications that do not require perfect crystallinity, such as energy storage materials and sensors.

An interesting variant is self-assembly, where materials organize themselves into ordered structures based on their chemical properties. Amphiphilic molecules with both hydrophobic and hydrophilic regions, for example, can guide the assembly of 2D materials into layered structures with controlled spacing and composition. This method uses nature's tendency toward order to develop complex architectures without directly manipulating each component.

For creating intercalated compounds, **electrochemical and chemical intercalation methods** are used. In these processes, guest species (ions or molecules) are inserted between host layers through chemical reactions or applied voltages. A classic example is lithium-ion batteries, where lithium ions move between graphite anodes and layered cathode materials during charge and discharge cycles. Similar principles are being applied to produce new 2.5D structures by intercalating not just ions but also molecular species that can alter electronic, optical, or mechanical properties.

Finally, techniques like **atomic layer deposition (ALD)** and **molecular beam epitaxy (MBE)** offer atomic-scale precision for creating highly controlled 2.5D structures. ALD builds materials one atomic layer at a time through self-limiting surface reactions, allowing for precise thickness control and conformity. MBE grows crystals by slowly depositing elements in an ultra-high vacuum environment, enabling atomic-level control of composition and structure.

Each fabrication approach provides a unique balance of precision, scalability, and material compatibility. The future of 2.5D materials will likely involve combining these techniques—using high-precision methods to create critical interfaces while applying more scalable approaches for less demanding components.

Material Properties: The dimensional crossover in 2.5D materials produces a wide range of properties that can be adjusted from quantum to classical regimes. This section examines how different layer arrangements create materials with tailored characteristics across various physical domains. The **electronic behavior** in 2.5D materials goes beyond the limits of both 2D and 3D systems. By designing band alignments between layers with different electronic structures, researchers have developed materials with specific band-gaps. The ability to adjust band-gaps is crucial in modern semiconductor design, as it allows for diverse applications such as highly efficient solar cells and advanced high-performance transistors.

For example, a vertical heterostructure combining graphene (with a zero bandgap) and MoS (with a 1.8 eV bandgap) creates a tunable Schottky barrier, enabling efficient photodetection over a wide spectral range. Researchers can further modify the interface properties through electrical gating or strain, allowing dynamic control over electronic behavior. This is different from conventional semiconductors, where bandgaps are fixed by chemical composition and require complex alloying for adjustments.

The **mechanical properties** of 2.5D materials offer additional design flexibility. While 2D materials like graphene have remarkable in-plane strength (stronger than steel despite being just one atom thick), they have difficulty with out-of-plane deformations. In contrast, 2.5D structures can be designed to have both high strength and controlled flexibility. For instance, MXene-polymer composites have alternating layers of rigid 2D sheets and flexible polymer chains, resulting in materials whose mechanical properties can shift from brittle to elastic by changing layer spacing and interfacial bonding. These materials can endure thousands of bending cycles without damage, making them essential for flexible electronics, wearable devices, and structural components that require both strength and flexibility.

What's particularly interesting is the **anisotropic thermal management** possible with 2.5D structures. In these materials, heat flow follows distinct paths based on direction: rapid conduction occurs along planes (where covalent bonds create efficient thermal pathways) while transfer between layers is limited (as weaker van der Waals forces hinder phonon transport).

This directional heat flow has been used in thermal interface materials for electronics cooling. Here, boron nitride-polymer composites effectively channel heat away from sensitive components while preserving electrical isolation. By managing layer spacing and orientation, thermal conductivity can be fine-tuned across several orders of magnitude—a level of control that is not achievable in traditional materials.

The **optical response** of 2.5D materials shows their dimensional complexity. When light interacts with these structures, it encounters environments that change dramatically over nanometer distances. This presents opportunities for precise manipulation of light-matter interactions.

In quasi-2D perovskites, for example, alternating organic and inorganic layers create quantum wells that confine excitons (bound electron-hole pairs). This results in sharp optical absorption and efficient photoluminescence. By adjusting the thickness of these layers, the emission wavelength can be varied across the visible spectrum. These materials have already transformed light-emitting diodes and show potential for next-generation display technologies.

The surface and interface chemistry of 2.5D materials offers more design options. Since a significant portion of their atoms is at surfaces or interfaces, these materials have increased chemical reactivity compared to their bulk counterparts. This characteristic can be utilized for sensing and catalysis applications.

For instance, MXenes, with their transition metal surfaces and customizable terminations (such as −O, −OH, −F), act as highly selective gas sensors and effective catalysts for hydrogen evolution reactions. The interlayer galleries in these materials can be engineered to selectively allow certain molecules while blocking others, thus creating molecular sieves with atomic-scale precision. Together, these diverse properties make 2.5D materials uniquely versatile platforms for tackling technological challenges in various areas, including energy, electronics, sensing, and structural application

Applications: Solutions Between Dimensions: The unique properties of 2.5D materials are leading to practical uses that are moving from labs to commercial settings. This section shows how these dimensional hybrids tackle important technological issues.

Next-generation electronics is one of the most immediate areas of application. As traditional silicon technology hits fundamental physical limits, 2.5D materials provide ways to break through these obstacles. Van der Waals heterostructures allow transistors to have atomically thin channels, nearly perfect interfaces, and adjustable electronic properties.

A notable example is the creation of tunneling field-effect transistors (TFETs) using vertical heterostructures made of graphene, h-BN, and graphene. These devices use quantum tunneling across the h-BN barrier, with precise control by gate voltages. This results in switching behavior that goes beyond the thermal limits of silicon transistors, potentially allowing for logic operations with much lower power, addressing one of computing's major challenges.

In **optoelectronics and photonics**, 2.5D materials mix strong light-matter interactions with adjustable electronic structures. This enables devices like photodetectors to selectively respond to specific wavelengths based on the bandgaps of the layers involved. For instance, a graphene/WSe_2/graphene structure can develop a photodetector with response times under 5 picoseconds, which is thousands of times faster than traditional semiconductor detectors, while still showing high sensitivity across a wide range of wavelengths.
Among the most impressive developments are in light emission. Quasi-2D perovskite materials, which have quantum-well structures, achieve photoluminescence quantum yields over 90%.

The energy sector has started using 2.5D materials for energy storage and conversion. Their layered designs serve as excellent platforms for battery electrodes, where ions move between layers during charging and discharging. For example, vanadium oxide-graphene composites create battery cathodes that have outstanding capacity and cycling stability, solving the degradation issues that often affect traditional materials In catalysis, MXenes and layered double hydroxides offer surfaces with precisely arranged active sites. These materials can facilitate reactions such as hydrogen production, oxygen reduction, and CO_2 conversion with efficiencies similar to precious metal catalysts but at a much lower cost. Their layered structure maximizes surface exposure while keeping electrical conductivity high, which is critical for electrocatalytic uses.

The field of **sensing and environmental monitoring** also benefits from the high surface-to-volume ratio and adjustable chemistry of 2.5D materials. Functionalised MXene sheets, for example, can detect volatile organic compounds at parts-per-billion levels by measuring changes in electrical conductivity when target molecules stick to their surfaces. The selectivity of these sensors can be tailored by altering surface terminations or adding molecular recognition elements between layers.

Water purification is another promising application, with graphene oxide membranes showing excellent selectivity in molecular sieving. By carefully controlling the spacing between layers through chemical cross-linking or intercalation, these membranes can block contaminants while allowing water molecules to pass through, potentially transforming desalination and water treatment methods.

Looking to the future, **quantum technologies** may turn out to be the most groundbreaking area for 2.5D materials. Their ability to host and manipulate unique quantum states—from topological edge modes to interlayer excitons—offers platforms for quantum information processing. Moiré superlattices in twisted van der Waals heterostructures, with their greatly adjustable electronic properties, present promising ways to create and control qubits, the basic units of quantum computers.

These varied applications demonstrate how 2.5D materials are connecting fundamental nanoscale phenomena with real-world technological solutions to some of society's biggest challenges.

Challenges and Future Outlook - Navigating the Dimensional Frontier: The journey from lab discovery to technological breakthrough is seldom straightforward. 2.5D materials face several key challenges before we can realize their full potential. This section explores these obstacles and the research efforts aimed at overcoming them.

One of the biggest challenges is scalable synthesis with atomic precision. While techniques exist to create high-quality 2.5D structures in the lab, producing them with consistent properties over large areas is still a challenge. For example, the twist angle in moiré super-lattices must be controlled to within a fraction of a degree to achieve the desired quantum effects. This level of precision is hard tomaintain during large-scale production. Promising strategies include epitaxial growth on engineered substrates that guide the desired orientation, automated assembly systems using computer vision and robotics for precise layer alignment, and self-limiting growth processes that naturally produce uniform structures. Recent advancements in chemical vapor deposition (CVD) have shown graphene/h-BN heterostructures with controlled twist angles over centimeter-scale areas, which is a promising step toward scalability.

Another significant challenge is **environmental stability and interface degradation**. Many 2.5D systems are sensitive to oxygen, water vapor, and other environmental factors that can change their properties over time. Additionally, the interfaces between different materials are vital to their unique functionalities, but they can degrade due to atomic diffusion or chemical reactions.

Tackling this issue involves **understanding degradation mechanisms and developing effective encapsulation strategies**. Research on atomically perfect encapsulation layers, self-healing interfaces, and stable passivation methods is progressing quickly. For instance, atomically thin h-BN has proven to be an excellent encapsulation material that protects the properties of sensitive 2D materials while providing a barrier against environmental contaminants.

The characterization of complex interfaces is another major hurdle. Traditional techniques often struggle to resolve the atomic-scale features that influence 2.5D behavior, especially at buried interfaces between layers. Developing non-destructive methods to visualize and measure these critical interfaces is essential for both fundamental understanding and quality control in manufacturing.

Recent advances in techniques such as **cross-sectional scanning, tunneling microscopy, low-energy electron microscopy, and synchrotron-based X-ray methods** are starting to provide detailed views into these hidden interfaces. Additionally, machine learning approaches are being developed to extract interface information from indirect measurements, potentially allowing for real-time monitoring during fabrication.

From a theoretical standpoint, predictive modeling across different scales remains tough. The multiscale nature of 2.5D materials which connects atomic interactions, quantum effects, and macroscopic properties—puts a strain on conventional simulation approaches. First-principles calculations become very computationally demanding for realistic system sizes. Meanwhile, continuum models overlook crucial quantum effects.

The likely solution lies in multiscale modeling frameworks that link methods across different lengths. This includes density functional theory for electronic structure, molecular dynamics for atomic motion, and finite element methods for macroscopic properties. Machine learning potentials that capture quantum effects while allowing for large-scale simulations are showing particular promise in this area.

Lastly, **integrating 2.5D materials with existing technology platforms** poses both technical and economic challenges. Incorporating these materials into established manufacturing processes requires compatibility with current equipment, materials, and protocols. The semiconductor industry, which has heavily invested in silicon technology, is especially resistant to disruptive changes.

The most effective near-term strategy seems to be finding niche applications where 2.5D materials provide significant advantages that warrant the development of specialized production methods. As manufacturing experience grows and costs decrease, broader use becomes more feasible. This pattern has historically been observed with new materials technologies, from carbon fiber composites to silicon carbide electronics.

Despite these hurdles, the direction of 2.5D materials research points toward an exciting future. The convergence of advancements in synthesis, characterization, theory, and device engineering is speeding up progress on multiple fronts. As we become better at manipulating matter at the atomic scale, the lines between basic science and practical technology continue to blur.

The ultimate promise of 2.5D materials lies not just in small improvements to existing technologies but in enabling entirely new capabilities. This includes quantum computers that work at room temperature, artificial photosynthesis systems that efficiently convert sunlight into fuel, neural interfaces that easily connect to biological systems, and many other innovations that we can hardly envision today.

As we delve deeper into this dimensional frontier, this space between the flatland of 2D and the bulk world of 3D, we will likely discover not only new materials but also new principles for engineering matter itself. The emergence of 2.5D materials may ultimately be seen not just as a minor development in materials science but as the start of a new chapter in our technological evolution.

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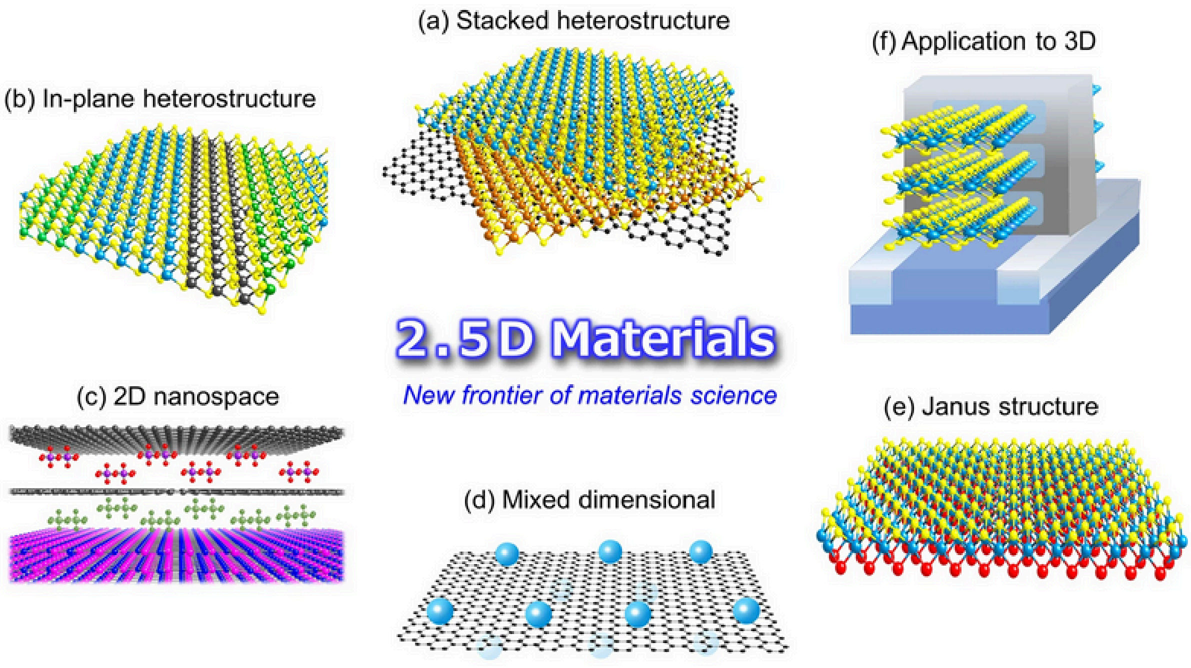
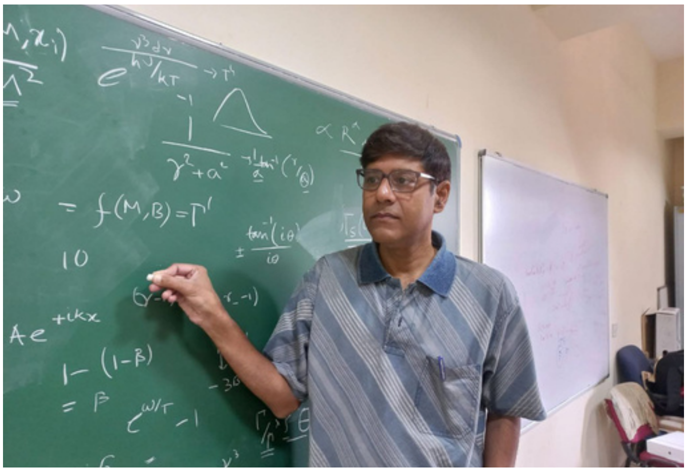


Various types of 2.5D materials are available, including multi-component stacked (a) and in-plane b heterostructures, intercalations (c), combinations with other dimensional materials (d), functionalizations (e), and 3D architectures with 2D materials (f)

EXTRATERRESTRIAL LABORATORIES, PRECISION, AND INTERDISCIPLINARY RESEARCH

Abhimanyu Ambikapathy, Gunda Sai Vinay

Prof. Banibrata Mukhopadhyay is a faculty member in the Department of Physics at the Indian Institute of Science. His research lies at the intersection of astrophysics, relativity, cosmology and astroparticle physics, with a focus on understanding extreme cosmic phenomena through theoretical modeling and observational interpretation. In this conversation, we explore his research insights, interdisciplinary approach, and perspectives on navigating the challenges and opportunities in scientific research.



Prof. Banibrata Mukhopadhyay

Let's begin the interview. Professor, Can you give us a brief overview about your current work ?

The main aim of my work lies in understanding and exploring relativistic astrophysics and laws of gravity on large scales using the basic norms of physics. As we know, physics is everywhere, but understanding of various phenomena, be it in the normal laboratories in our Earth, all the way to the planets and beyond, which can also be considered as "extra-terrestrial laboratories" is quite important. Let's take superconductivity as an example. It has been witnessed in our labs here, but high-density superconductivity can only be understood in extraterrestrial objects. So, to answer the question, my work requires understanding physics in the "exotic" sense, which requires exploring space-based objects and fields, as they show behavior not obvious to what we see here on the surface of the Earth.

You had mentioned "extra-terrestrial laboratories". How precise are the measurements required in the observations for understanding the astrophysics of these celestial bodies ?

Precision is an important pillar when it comes to interpretation of experimental data. The way I understand is at first, we formulate a model keeping a broad application in mind. This model should be applicable at some parameter regime with certain conditions in accordance with terrestrial physics. But the model should be in such a way that with the parameters we have at our disposal, we can do necessary tweaks so that they simulate the dynamics of extra-terrestrial physics. Now, light is one of our primary carriers of information in the universe. So, to understand extra-terrestrial physics, we tend to look at light arriving from distant sources, say neutron stars. Thus, the bottom line of my proposal is that the same model at different regimes should be capable to explaining the Earth-based to extra-terrestrial experiments. To answer this question, our important consideration is on the uncertainties arising in the measurements and the subsequent impact it can have on theory and the model. The main constraints one should set for a hypothesized model are minimum number of parameters and behaviors (and/or variations) of said parameters in different regimes. The other significant part of this is the target body which you are evaluating for your study. As for the model, there arises two important questions: One, whether we are formulating the behaviorally appropriate model or not and two, whether our model is in accordance with source-related discrepancies, which in the case of light or any wave in general, include distortion and scattering due to other bodies and possible attenuation.

This might cloud some properties of the original physics; hence we need to be careful with these things while observing. One has to consider and develop precise methodologies which can account for errors observed both theoretically and practically, where often the model predicted error could be less than the observational uncertainties.

Building on the question of "Extra-terrestrial labs", can you give us examples of celestial objects on which your work is focused ?

Since my work involves different fields of physics and at times, their interface, I have changed fields quite a number of times throughout my research career. But primarily, my work involves exploring the behavior of compact objects, which include black holes, neutron stars and white dwarfs to name some examples. The study of the associated fluid mechanics of these bodies also comes under this, with the premise being matter following a fluid like behavior because of gravity, and it is this fluid which produces light that we are analyzing and understanding.

Since this moves onto astrophysical fluid mechanics, we now try to learn and find certain similarities between the fluid present "around" compact stars and the ones we see in laboratory experiments. The target goes hand in hand with the model we tend to formulate from this. My main aim is to try and unify the laboratory and astrophysical fluid problem(s) and so, to answer your question, the research involves compact objects, the astrophysical fluid along with some applications of very early cosmology. The energy scales for early universe were of the order of 10^{16} GeV along with things related to the Grand Unification Theory; this was for believing that these epochs did exist, but we don't have direct solid proof yet, thus making the physics of the early universe too a topic of my interest.

Based on your publications and research, one can say that a significant part of it involves the interface of two or more areas of physics and/or the sciences in a broader sense. Can you give walk through the process a researcher undertakes when he/she approaches a problem which combines 2 or more fields, subsequently people associated with them as well ? How do you go about exploring the problem and working out the details ?

Interface is an important part, yes. For example, Cosmology is used alongside astrophysical part and that requires statistical physics. Also, we need gravitational physics, Field Theory (multiple disciplines). But the problem remains the primary target. Since we have been discussing about the analysis of light in the previous questions, let's continue in the same direction.

Now, when it comes to light, we have to look at the radiative processes behind it, mainly in the region of the source where it can be influenced by strong gravity. With gravity affecting the fluid behavior of the underlying systems which produce light, we have to look at the photon behavior in strong gravity and here the next field comes up. Also, we believe that the observable universe is spatially flat but in near massive bodies, the spacetime region is curved which is due to the strong

gravitational interactions in the first place. This puts General Relativity into the mix as well. Also, the light we analyze is always not purely just a photon stream; it is more of a complex radiation which can have other high energy particles, like neutrinos for example. Thus, particle physics in curved spacetime is applied here, which too requires statistical mechanics. We can see how, as we go deeper into a certain problem, the diverse fields come into picture, one by one. What matters is how you combine them and use the associated theories and information from each one properly at required parts of the problem you are working on.

I would like to give a real-life example to illustrate the answer. One of my students in the last couple of years has been interested in the field of neuroscience. Even though from the outset it doesn't look much physics is involved (with respect to compact objects), certain techniques and laws, like non-linear dynamics which we use regularly are being used to understand the working of our brain signals. She had contributed to two papers with me before embarking completely into neuroscience. Some concepts like the Bekenstein bound, which we apply in thermodynamics, are also useful in neuroscience, particularly in the study of information processing, brain efficiency, and consciousness. As we go deeper into the field of research, we can see that in a broader sense of speaking, all of it is just science and just the different parts of it that we combine, apply and learn.

However, interpretation and bias are important parameters to keep in mind. For example, in the field of cosmology, the main theory as to how things started, there still exists a small divide between those who advocate the Steady State Theory and the others, who support the Big Bang Theory, which we have now come to terms with, theoretically and observationally. The Big Bang Model, when proposed initially, did have lots of loopholes and needed several corrections and tweaks. Now, it has evolved with time, with refinement and backing of experiments and observations to what it is we see and learn today. The same could be said for the progress in the field of Dark Matter research.

We have heard a lot of theories about dark matter in the recent times, like whether it is a fluid and subsequently leading to particle behaviour. If so, what are the fundamental particle(s) that constitute DM, is it stuff like primordial black holes, axions, neutrinos or something else. Another theory that pops up in the same context is whether Dark Matter does NOT actually exist and it's just a modification of Newtonian gravity and potential. What is your perspective on it?

Due to lack of direct proof to a certain extent, Dark Matter and its associated perspectives remains a tricky field to navigate through. Even now, despite having conclusive proof of the otherwise, MOND (Modified Newtonian Dynamics) is one such theory which still hasn't been completely ruled out when it comes to explaining Dark Matter. Also, many of the problems related to the problems in space which are satisfied with by the presence of DM are explained by MOND, this warranting their significance, albeit to the limit of its correctness.

A part of my work revolves around Primordial Black Holes and Hawking radiation which is linked with dark matter. There was a good student of mine, one of your super-seniors who had presented on the same for his bachelor's thesis. So yeah, while we do have several candidate particles/entities, for answering what Dark Matter is made of, we still haven't singled out on one of them, which satisfies all the required criteria and can possibly account for all the observed effects shown by DM.

When Dark Matter first entered the scene of cosmology and astrophysics, it was used more like a fit into the equations to work around the discrepancies. So, how did it start its entry and effect into your work and in the field of relativistic astrophysics?

It entered at quite an early part of my career, I'd say. I remember, when I was a student myself, seeing the MOND theory being embraces as the model for dark matter. But over the years, the number of followers of "real" dark matter i.e. people supporting the existence of an actual presence of a new form of matter, not just a modified potential-influenced gravitational field, increases. Although MOND was quite a powerful idea at least till the early 2000s, the scene changed soon after, and it was further backed by the rise of Gravitational Wave Astronomy. Occasionally, people used to think of possible modifications to Einsteins theory of gravity and now that too has been on the increase. The primary merit of this is how it facilitates in explaining the observations related to how DM is affecting the dynamics and behaviour of universe, for example cluster mergers, galaxy rotation curves and to an extent, density fluctuations of the early universe. I hope that answered your question.

You have also worked a bit in Quantum Mechanics. Can you give us a brief account of the same?

Yes, like I said at the beginning, my work isn't strictly limited to astrophysics, I try to understand the physics and the behaviour of exotic compact objects, with the help of norms of fundamental physics. Quantum physics, being one of the said fundamental norms, is quite important and needed to understand certain problems of nuclear astrophysics. For example, fusion reactions; Protons are repulsive to each other due to the Coulomb force, but they still interact and fuse to create energy. This can be explained in the QM world. The more niche part which is used when it comes to figuring out observations and more is Relativistic Quantum Mechanics which is intricately linked with Astrophysics. Like we had discussed in an earlier question, we are looking at the whole of science in the grand scale, not just astrophysics and it comes down to which fields we combine and apply and how we do it meticulously.

"It is through observation and verification that a theory transforms from speculation into meaningful understanding of nature."

Around a year back, some of my students had arranged a conference commemorating my work across the last 25 plus years, where we had lots of wonderful discussions in different areas I had worked on, with my students and my colleagues. A book by Springer will be published, containing summarized articles where I've explained the work I've done in different fields to good detail.

Since the work you're pursuing right now requires observational data and evidence across the cosmological scales, lot of collaborations are involved. Are you currently working alongside any experimental lab/group? If yes, could you tell us more about it?

Oh yes, experimental collaborations are an important arm of the branch of work I'm currently in. One immediate example I can give is ASTROSAT,which is a multi-wavelength space telescope. One of the main parts of the EM spectrum used in observational data in the X-Ray wave portion captured by the probe. The theory I make should be able to explain what we've observed and interpreted from the x-ray data obtained from the target region being looked at. It is through observation and verification that a theory transforms from speculation into meaningful understanding of nature.
As for other examples, I've been working on white dwarfs and neutron stars, so there I had to use the data gathered by probes like Gaia by the European Space Agency. The thing with cosmological data obtained these days is that we look at it as a huge puzzle. We need to interpret them in accordance with our experiment, bearing in mind the constraints we have set for the target problem and/or the influence of other celestial entities, up to and including Dark Matter and Dark Energy. Also, we have and tend to believe on the different possible models for the universe, which again are backed by observational data, and its behviour. Take expansion, for instance. Big Bang cosmology based models give us the Hubble constant and the scale factor from which it's derived, showing us the rate of expansion of universe. The data from Supernovae further tells us that this scale factor is time dependent, showing us the that the expansion of the universe is indeed accelerating. Initially, the value of H proposed by Edwin Hubble himself was found to be vastly off to what we have today based on corrected calculations and experiments. So yes, that's how deep rooted the collaborative effort of observations lie, in the postulation of theories and results.

Sir, when we are dealing with the cosmological scales, we usually tend to introduce, at least to a certain extent, a degree of homogeneity and isotropy. But as we go further and deeper, we get the fluctuations and associated anisotropies we anticipate as the cause for several effects. So, when working with certain problems, how do you get around these things, with respect to equations and the math framework?

The models proposed in cosmology are made so, bearing in mind the fact that our observable universe shows behaviour of a spatially flat and locally isotropic and homogenous one. But the main thing in consideration is the target body or region in space which is the focus of our research study problem. Indeed, there are observational signature which we pick up on as we delve deeper into our problem and that's where we get the non-uniformities. This warrants the need to consider some sort of perturbative effect at play. Taking Red-Shift as an example, we look at trends like the red-shift with respect to galaxy speeds, distance (radial or co-moving) and more. For our assumptions, we generally take zero curvature for our universe but, to understand the origin of gravitational waves from the events of the order of a billion years, we MUST include curvature because that was how our universe was. Basically, we can visualize gravitational waves as the ripples in the spacetime curvature caused due to gravitational events (in the present universe, e.g., the merger of black holes). The bottom line, to answer the question, is that we usually consider the early universe to be isotropic and homogeneous. But, to appropriately detect and analyze our data in the context of the experiment or problem we're working on, we must choose to include or leave the details at the designated parts, accordingly.

This might be a question away from the theories and things we've discussed till now, Sir. Do you have any advice and guidance tips for an undergraduate trying to embark on a research career? Especially when they're trying to navigate through the ups and downs encountered in the initial stages and getting to stay focused and progress through it.

Now that you guys have reached the end of your 2nd year, you must have a fair bit of exposure to reading projects and short stints of research work in an introductory level. I wouldn't say that up to now there would've been much of ups and downs at a significant level, only for maybe some moments

“So, pre-deciding on your career path without exploring other areas may not be ideal in the general perspective.”

where you couldn’t understand and figure out some certain concept while working in the project or during the semester. With time and experience, you would be facing much of these moments. My suggestion to students of your age group and in general, those who are starting on a career in research would be to explore a lot of fields and get adequate exposure to understand the working and choose a particular path to pursue. Eventually, by the time of your completion of the UG degree, you will have some idea of what you can do further down the road based on your interest. I am interested in astrophysics and gravity, but I have worked on other fields too and at times, more than one for certain problems. So, pre-deciding on your career path without exploring other areas may not be ideal in the general perspective.

One of the important things that a student can do for the same is to approach and interact with professors and seniors who have been in the field for a while. And with being in IISc, where a lot of researchers working on various areas of science in their own specialised depths are available, opportunities to network and learn are in plenty.

Basically then, the first thing to do is to take up one area or field and start doing a small project in it, like an introductory reading under a professor. If you’re able to clock your interest in it in a healthy way, it’s good to pursue the project and subsequently, know more about the field.

But if it’s the other way around, try exploring more and find other areas to which you can intellectually migrate to, from the one you’re in now. These projects and more work you do along with your academics will be influential in deciding the area you would want to work on for the thesis. I have seen some students who have continued work in the same field for their master’s thesis right after their UG degree. The key lies in interacting with your professors and fixing on your goal and your ideas in line. At a young age, we usually tend to have wrong ideas and beliefs about certain concepts, but these get clarified with more learning and experience.

Also, we should not ignore certain areas of physics and science, in general just because of our notions and beliefs, they will come in handy someday and might be pivotal. For example, I didn’t take up CMP initially but after reading and working a bit, I realised its importance in understanding compact objects and more when it comes to our universe. Like, the centre of neutron stars, which is superconducting. We have found that although the very core of the neutron star is not superconducting, the surrounding region to a certain limit, does show the property. So yes, this is one application of CMP into the field of physics of “exotic” objects.

“Also, we should not ignore certain areas of physics and science, in general just because of our notions and beliefs, they will come in handy someday and might be pivotal.”

CAN CURVED SPACETIME FLIP A NEUTRINO? WHEN GRAVITY BREAKS CPT SYMMETRY

Gunda Sai Vinay

Introduction:

Neutrinos occupy a unique position in modern physics: they are among the most abundant particles in the universe, yet among the least understood. Once thought to be massless, they are now known to possess tiny but nonzero masses, inferred from the discovery of neutrino oscillations, the phenomenon in which neutrinos spontaneously change flavor as they propagate. This behavior, first observed in solar and atmospheric neutrino experiments, revealed that neutrinos of definite flavor (electron, muon, and tau) are not the same as those of definite mass. Instead, each flavor state is a superposition of several mass eigenstates, which evolve differently with time. The interference between these components leads to the periodic conversion of one flavor into another, an effect well described by quantum mechanics in flat spacetime.

Conventional neutrino oscillations preserve lepton number and are fundamentally a consequence of mass differences between the neutrino eigenstates. A neutrino born as an electron neutrino remains a neutrino, never transforming into an antineutrino, although its flavor may change. This process assumes that the underlying symmetries of nature, including CPT invariance, remain intact.

CPT symmetry, a central theorem in local relativistic quantum field theory, asserts that the combined operations of charge conjugation (C), parity transformation (P), and time reversal (T) leave all physical laws invariant. Among its many implications is the equality of masses and lifetimes for particles and their corresponding antiparticles. Any observation of CPT violation would, therefore, signal physics beyond the Standard Model and perhaps point toward an interplay between quantum fields and spacetime geometry.

CPT : CPT symmetry is a fundamental principle in quantum field theory, stating that the laws of physics remain invariant when three fundamental transformations i.e., Charge conjugation (C), Parity transformation (P), and Time reversal (T) are applied simultaneously. Neutrinos occupy a unique position in modern physics, they are among the most abundant particles in the universe, yet among the least understood. Once thought to be massless, they are now known to possess tiny but nonzero masses, as inferred from the discovery of neutrino oscillations, the phenomenon in which neutrinos spontaneously change flavor as they propagate. This behavior, first observed in solar and atmospheric neutrino experiments, revealed that neutrinos of definite flavor (electron, muon, and tau) are not the same as those of definite mass. Instead, each flavor state is a superposition of several mass eigenstates, which evolve differently with time. The interference between these components leads to the periodic conversion of one flavor into another, an effect well described by quantum mechanics in flat spacetime.

In quantum field theory, the three fundamental discrete symmetries are defined as follows: C, which swaps particles with their antiparticles; P, which inverts spatial coordinates (like viewing the universe in a mirror); and T, which reverses the direction of time. According to the CPT theorem, any Lorentz-invariant local quantum field theory with a Hermitian Hamiltonian must obey CPT symmetry. This implies that a process and its CPT transformed counterpart, where all particles are replaced by antiparticles, spatial coordinates are inverted, and time runs backward—occur with identical probabilities. Experimental tests, such as comparing the properties of particles and antiparticles (for example, electrons and positrons), have so far confirmed CPT symmetry to an extraordinary degree of precision. A violation of CPT symmetry would have profound implications, signaling a breakdown of fundamental assumptions about spacetime and quantum theory itself.

In Curved Spacetime : In curved spacetime, however, the situation may differ. When fermions such as neutrinos propagate through a gravitational field, their spins interact with the curvature of spacetime via the spin connection. This coupling modifies the Dirac Lagrangian by introducing an additional interaction term proportional to an axial vector field, often denoted B_a, which arises from the geometry itself. The corresponding term in the Lagrangian takes the form

$$\mathcal{L}_I = \bar{\psi}\gamma^a\gamma^5 B_a\psi$$

where $\bar{\psi}\gamma^a\gamma^5\psi$ is the axial current of the fermion. Physically, this current distinguishes between left- and right-handed components of the spinor field, it changes sign under parity transformation and therefore couples differently to particles and antiparticles. In curved spacetime, the effective axial field B_a represents a pseudovector that encapsulates how the local inertial frames twist and rotate due to curvature or rotation of the background geometry.

Because the axial current $\bar{\psi}\gamma^a\gamma^5\psi$ is CPT-odd, whether or not the interaction $\mathcal{L}_I$ preserves CPT depends on the behaviour of B_a under spacetime inversion. If B_a is CPT-even, meaning it remains unchanged under the combined reversal of space and time coordinates, then the product $\bar{\psi}\gamma^a\gamma^5 B_a\psi$ changes sign and thus violates CPT symmetry. This mechanism provides a natural, geometric origin for CPT violation, one that requires no modification of fundamental interactions but arises directly from the curvature of spacetime. Such a coupling has profound implications for neutrinos. Since neutrinos are chiral particles, left-handed neutrinos and right-handed antineutrinos, the axial term affects them asymmetrically. In regions of strong gravitational curvature or rotation, the effective energies of neutrinos and antineutrinos could differ, even if their rest masses are identical. This asymmetry, encoded in the gravitational field B_a, opens the possibility for neutrino–antineutrino oscillation, a process forbidden in flat spacetime under exact CPT and lepton-number conservation. The exploration of this gravitationally induced effect forms the foundation of the work by Banibrata Mukhopadhyay in 2007, where he demonstrates how the axial coupling of spin to curvature can naturally give rise to CPT and lepton number violation, leading to neutrino–antineutrino oscillations in anisotropic or rotating spacetimes.

The Axial Coupling in Curved Spacetime :

In flat spacetime, the dynamics of a spin-$\frac{1}{2}$ particle are governed by the Dirac equation,

$$(i\gamma^a\partial_a - m)\psi = 0$$

where γ^a are the gamma matrices and m is the rest mass. This equation assumes a global inertial frame in which space and time coordinates are uniform everywhere. However, in curved spacetime such global inertial frames do not exist, the local basis vectors continuously rotate relative to one another as dictated by the curvature. To describe fermions under these conditions, the Dirac equation must be generalized by introducing a covariant derivative that accounts for the curvature-induced rotation of local frames.

The tetrad formalism : The Dirac equation (and any spinor theory) is originally formulated in flat Minkowski space, which uses constant gamma matrices γ^a satisfying

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}$$

where η^{ab} is the Minkowski metric.

On a curved spacetime, the metric $g_{\mu\nu}(x)$ varies with position, so one cannot directly use these flat-space matrices γ^a. We therefore need a way to connect the curved coordinate basis to locally flat Minkowski frames. To incorporate curvature while preserving the local Lorentz symmetry of the Dirac field, one introduces a set of tetrads (or vierbeins) e_a^{μ}, which form a bridge between the curved spacetime described by the metric $g_{\mu\nu}(x)$ and the locally flat Minkowski frame described by η^{ab} :

$$g_{\mu\nu} = e_\mu^{\ a} e_\nu^{\ b}\eta_{ab}$$

Here, the indices a, b, c, . . . refer to local Lorentz coordinates, while $\mu, \nu, \ldots$ denote the general spacetime coordinates. The tetrads allow one to define locally flat frames at each point in spacetime, within which the gamma matrices satisfy the usual anticommutation relation $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$.

The derivative acting on a spinor must now include a correction term, called the spin connection ω_{abc}, which describes how the local frames rotate as one moves through spacetime. The spin connection tells us how the tetrad basis changes under parallel transport. It plays the same role for local Lorentz indices that the Christoffel symbols $\Gamma^{\mu}_{\nu\rho}$ play for spacetime indices.

The spin connection is defined through the tetrad postulate :

$$\nabla_\mu e^a{}_\nu = \partial_\mu e^a{}_\nu + \omega^a{}_{b\mu}\, e^b{}_\nu - \Gamma^\lambda{}_{\nu\mu}\, e^a{}_\lambda = 0$$

The covariant derivative of a spinor is defined as

$$D_a = \partial_a - \frac{i}{4}\,\omega_{bca}\sigma^{bc}$$

where $\sigma^{bc} = \frac{i}{2}[\gamma^b, \gamma^c]$ are the generators of Lorentz transformations. The generalized Dirac equation then becomes

$$(i\gamma^a D_a - m)\psi = 0$$

Separation into vector and axial components : Expanding the covariant derivative reveals that the spin connection introduces new interaction terms between the spin of the particle and the curvature of spacetime. These terms can be organized into a vector part, which couples like a potential, and an axial (pseudovector) part, which couples to the chiral structure of the spinor. After suitable algebraic manipulations, one obtains an effective Lagrangian that separates the flat-space dynamics from the purely gravitational interaction:

$$\mathcal{L} = \sqrt{-g}\,\bar{\psi}\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi$$

Here, the first two terms correspond to the familiar flat-space kinetic and mass terms, while the additional term represents the spin–curvature interaction mediated by the axial vector B_a.

Definition and interpretation of B_a : The quantity B_a is given explicitly by

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e_c^{\ \lambda} + \Gamma^\lambda_{\alpha\mu} e_c^{\ \alpha} e_a^{\ \mu}\right)$$

where ϵ^{abcd} is the antisymmetric Levi-Civita symbol, and $\Gamma^\lambda_{\alpha\mu}$ are the Christoffel symbols representing the gravitational connection in coordinate space. This expression shows that B_a arises entirely from geometric quantities: the derivatives of the tetrads and the affine connection. It vanishes in flat spacetime, where tetrads are constant and the connection coefficients are zero, but becomes nonzero when the spacetime has curvature or rotation.

Physically, B_a represents the axial part of the spin connection, or equivalently, the component of the spacetime connection that couples to the spin's orientation. It acts like a background pseudovector field, a purely geometric analogue of an axial magnetic field, that distinguishes between left-handed and right-handed spin states. The presence of B_a therefore implies that spacetime curvature can induce chiral asymmetry in fermionic propagation.

The axial current and CPT properties : The interaction term involving B_a couples this geometric pseudovector to the axial current

$$J_5^a = \bar{\psi}\gamma^a\gamma^5\psi$$

This current, unlike the ordinary vector current $\bar{\psi}\gamma^a\psi$, changes sign under parity and time reversal, and is thus a pseudovector. Its coupling to B_a allows the geometry of spacetime to act differently on left and right handed components of the spinor field. The behavior of $\mathcal{L}_I = J_5^a B_a$ under CPT transformations depends on the nature of B_a. If B_a changes sign when space and time are inverted, i.e. it is CPT-odd, then the product $J_5^a B_a$ is CPT-even, and the interaction preserves CPT symmetry. However, if B_a is CPT-even (remaining unchanged under reversal), then the interaction term becomes CPT-odd, resulting in explicit CPT violation. In the latter case, neutrinos and antineutrinos experience different effective gravitational couplings, which lead to distinct energy dispersion relations.

This mechanism provides a purely geometrical route to CPT violation: the symmetry breaking arises not from a new fundamental interaction, but from the intrinsic properties of the spacetime background. As Mukhopadhyay emphasizes, B_a is zero in spherically symmetric or isotropic spacetimes, such as the Schwarzschild or Robertson–Walker metrics, but becomes nonzero in rotating or anisotropic geometries, such as the Kerr black hole or the Bianchi-type cosmological models. These are precisely the environments where one can expect gravitationally induced neutrino–antineutrino asymmetry to emerge.

Energy Splitting and Neutrino–Antineutrino Oscillation :

The axial coupling introduced in the previous section has a direct consequence for fermions such as neutrinos: it modifies their energy - momentum dispersion relation. Since the interaction term $\bar{\psi}\gamma^a\gamma^5 B_a\psi$ couples differently to the left and right handed components of the spinor field, it acts as an effective background field that distinguishes between neutrinos and antineutrinos.

Energy dispersion relations : In the presence of a nonzero gravitational pseudovector field B_a , the modified Dirac equation leads to distinct energy eigenvalues for the left and right chiral components of the spinor. These correspond respectively to neutrinos and antineutrinos:

$$E_\nu = \sqrt{(\mathbf{p}-\mathbf{B})^2 + m^2} + B_0$$

$$E_{\bar{\nu}} = \sqrt{(\mathbf{p}+\mathbf{B})^2 + m^2} - B_0$$

Even if the rest masses of the neutrino and antineutrino are identical, the presence of B_a introduces an energy splitting between them. The temporal component B_a contributes additively to the energy, while the spatial component **B** modifies the effective momentum. This splitting originates purely from the curvature or rotation of the background geometry, and therefore represents a gravitational analogue of the Zeeman effect, in which an external field differentiates between two spin states.

Physical interpretation : The above expressions imply that the propagation of neutrinos and antineutrinos is no longer identical in a curved or rotating spacetime. The energy difference,

$$\Delta E = E_\nu - E_{\bar{\nu}} \approx 2(B_0 - |\mathbf{B}|)$$

is directly proportional to the curvature-induced pseudovector components. In static or isotropic spacetimes, such as the Schwarzschild or Robertson–Walker metrics, these components vanish and $\Delta E = 0$, restoring CPT invariance. However, in anisotropic or rotating backgrounds, such as in the vicinity of a Kerr black hole or in the early anisotropic universe, $B_a \neq 0$, and neutrino–antineutrino asymmetry naturally arises.

This energy asymmetry has a clear physical meaning. Because neutrinos are left-handed and antineutrinos are right-handed, the axial coupling makes them experience opposite effective potentials in curved spacetime. The gravitational field thus lifts the degeneracy between the two, endowing them with slightly different propagation phases even when their rest masses are the same.

Formation of mixed states : To analyze the resulting oscillation phenomenon, one constructs linear combinations of the neutrino and antineutrino states that serve as the true propagation eigenstates. Analogous to the neutral kaon system, these can be written as

$$|m_1\rangle = \cos\theta\,|E_\nu\rangle + \sin\theta\,|E_{\bar{\nu}}\rangle$$

$$|m_2\rangle = -\sin\theta\,|E_\nu\rangle + \cos\theta\,|E_{\bar{\nu}}\rangle$$

where θ is an arbitrary mixing angle. The time evolution of these states is governed by the energy difference between $|E_\nu\rangle$ and $|E_{\bar{\nu}}\rangle$, which induces a relative phase as they propagate.

Oscillation probability : The probability of transition from one mixed state to the other after a time t is given by

$$P_{12} = \sin^2 2\theta \, \sin^2 \delta$$

$$\delta = \frac{(E_\nu - E_{\bar{\nu}})t}{2} = \left[(B_0 - |\mathbf{B}|) + \frac{\Delta m^2}{2|\mathbf{p}|}\right]\frac{t}{2}$$

When the neutrino and antineutrino masses are identical ($\Delta m^2 = 0$), the oscillation arises purely due to the gravitational term $(B_0 - |\mathbf{B}|)$. This is in contrast to conventional flavour oscillations, which require nonzero mass differences. Hence, gravity itself can induce neutrino–antineutrino oscillations even for mass-degenerate or massless neutrinos.

Oscillation length : The corresponding oscillation length, defined as the distance over which the oscillation phase completes one full cycle, is

$$L_{\text{osc}} = \frac{\pi\hbar c}{|B_0 - |\mathbf{B}||}$$

This quantity determines the spatial scale over which neutrino–antineutrino conversion is significant. The oscillation length is inversely proportional to the magnitude of the curvature-induced pseudovector field: strong gravitational curvature or rotation (large B_a) leads to rapid oscillations, while weak gravitational fields correspond to extremely long oscillation scales, rendering the effect negligible in terrestrial conditions.

Implications : The existence of an energy splitting between neutrinos and antineutrinos implies a violation of both CPT symmetry and lepton number conservation. The neutrino–antineutrino oscillation predicted by this mechanism is therefore a direct manifestation of the spin–curvature coupling encoded in B_a. The author emphasizes that such an effect would be significant only in regions of strong gravitational influence, such as near compact astrophysical objects (rotating neutron stars or black holes) or in the anisotropic early universe. In these environments, the curvature-induced CPT violation could influence the neutrino spectra, energy transport, and even the matter–antimatter asymmetry of the universe.

Astrophysical and Cosmological Implications :

The mechanism of gravity-induced neutrino–antineutrino oscillation becomes relevant in physical situations where the curvature or rotation of spacetime generates a nonzero axial pseudovector field B_a. While in static, spherically symmetric spacetimes the field vanishes identically, it becomes nonzero in geometries that are either anisotropic or possess rotational symmetry. Two particularly important examples are the Kerr spacetime, representing a rotating black hole, and the Bianchi type-II model, which describes an anisotropic early-universe geometry. In both cases, the presence of off-diagonal metric components gives rise to a nonvanishing B_a, leading to potential CPT and lepton-number violation.

Anisotropic early universe: Bianchi type-II model : In the early stages of cosmic evolution, the universe may have been anisotropic before isotropization set in during the radiation-dominated era. A simple representation of such a geometry is provided by the Bianchi type-II metric, given by

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & S(t)^2 & 0 & -S(t)^2 h(y) \\ 0 & 0 & R(t)^2 & 0 \\ 0 & -S(t)^2 h(y) & 0 & R(t)^2 f(y)^2 + S(t)^2 h(y)^2 \end{pmatrix}$$

where

$$f(y) = y, \quad h(y) = -\frac{y^2}{2}$$

The corresponding orthogonal set of non-vanishing tetrad (vierbein) components can be chosen as

$$\begin{aligned} e^0_t &= 1, \\ e^1_x &= \frac{f(y)R(t)S(t)}{\sqrt{f(y)^2R(t)^2 + S(t)^2h(y)^2}}, \\ e^2_y &= R(t), \\ e^3_z &= \sqrt{f(y)^2R(t)^2 + S(t)^2h(y)^2}, \\ e^3_x &= -\frac{S(t)^2h(y)}{\sqrt{f(y)^2R(t)^2 + S(t)^2h(y)^2}} \end{aligned}$$

The components of B_a are then

$$\begin{aligned} B^0 &= \frac{4R^3S + 3y^2RS^3 - 2yS^4}{8R^4 + 2y^2R^2S^2}, \\ B^1 &= 0, \\ B^2 &= \frac{(4yR^2 - 8RS - y^3S^2)(RS' - R'S)}{8R^4 + 2y^2R^2S^2}, \\ B^3 &= 0 \end{aligned}$$

where $R(t)$ and $S(t)$ are scale factors depending on cosmic time t. The off-diagonal component g_{xz} introduces an intrinsic rotation into the geometry, producing a nonzero coupling between the spin of the fermion and the curvature. For this metric, one finds that the components B^0 and B^2 are non-vanishing, while the others vanish. Importantly, these components are not CPT odd i.e. do not just change sign under CPT. Consequently, the product $J_5^a B_a$ changes under CPT, implying explicit CPT violation. The strength of B_a in this epoch depends on the anisotropy rate of the universe, and hence would have been significant only at very early times. The author suggests that such gravitationally induced CPT violation in the early universe could play an important role in generating a lepton asymmetry. The asymmetry between neutrinos and antineutrinos arising from the curvature - induced energy splitting could then be partially converted into a baryon asymmetry through electroweak sphaleron processes, which conserve B-L but violate B and L separately. This provides a natural geometric route to leptogenesis, and ultimately to the observed matter–antimatter imbalance in the universe.

Rotating black holes (Kerr geometry) : A second physical context in which B_a acquires nonzero values is the spacetime surrounding a rotating (Kerr) black hole. The Kerr metric, expressed in Boyer–Lindquist coordinates (t, r, θ, ϕ), is given by

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2Mr}{\rho^2}\right) & 0 & 0 & -\frac{2Mar\sin^2\theta}{\rho^2} \\ 0 & \frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\frac{2Mar\sin^2\theta}{\rho^2} & 0 & 0 & \left(r^2 + a^2 + \frac{2Ma^2r\sin^2\theta}{\rho^2}\right)\sin^2\theta \end{pmatrix}$$

where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$. Here, M is the mass of the black hole and 'a' its spin parameter.

The corresponding non-vanishing components of the tetrad (vierbein) are given as

$$\begin{aligned} &{e^0}_t = 1, \quad {e^1}_t = -\frac{\alpha}{\rho}s_1, \quad {e^2}_t = -\frac{\alpha}{\rho}s_2, \\ &{e^3}_t = -\frac{\alpha}{\rho}s_3, \quad {e^1}_x = 1 - \frac{\alpha}{\rho}s_1v_1, \\ &{e^2}_x = -\frac{\alpha}{\rho}s_2v_1, \quad {e^3}_x = -\frac{\alpha}{\rho}s_3v_1, \\ &{e^1}_y = -\frac{\alpha}{\rho}s_1v_2, \quad {e^2}_y = 1 - \frac{\alpha}{\rho}s_2v_2, \\ &{e^3}_y = -\frac{\alpha}{\rho}s_3v_2, \quad {e^3}_z = 1 - \frac{\alpha}{\rho}s_3v_3. \end{aligned}$$

from this, we obtain the component

$$B^0 = \epsilon^{ijk}\, {e_i}^{\lambda}\, \partial_j e_{\lambda\, k} = -\frac{4a\sqrt{M}\, z}{\bar{\rho}^2\sqrt{2r^3}}$$

where $\bar{\rho}^2 = 2r^2 + a^2 - x^2 - y^2 - z^2$

similarly others. We know that spin parameter changes sign upon CPT and so does z. Rest all remain invariant. Thus, B^0 remains invariant under CPT. This makes the whole lagrangian CPT asymmetric.

Thus, this causes energy difference between neutrino and anti-neutrino which in turn causes oscillations from neutrino to anti-neutrino and vice versa.

Magnitude of the effect and Consequences : Although the gravitationally induced splitting is extremely small in weak-field conditions, it can become significant in regions of strong curvature or rotation. The characteristic oscillation length

$$L_{\text{osc}} = \frac{\pi\hbar c}{|B_0 - |\mathbf{B}||}$$

can vary widely depending on the strength of the gravitational field. Estimates for various environments yield the following approximate values:

- Near rotating black holes : $B_a \sim 10^{-19} - 10^{-28}$ GeV giving $L_{\text{osc}} \sim 10 - 10^7$ km.
- In the early universe at the grand unification (GUT) scale: $B_a \sim 10^5\ GeV,$ yielding $L \sim 10^{-24}$ km.
- In terrestrial or atmospheric conditions: $B_a \sim 10^{-37}$ GeV, corresponding to $L_{osc} \sim 10^{-24}$ km.

One of the significant consequences of this phenomenon in rotating black holes lies in its impact on r-process nucleosynthesis. Supernovae are believed to be the primary astrophysical sites where the r-process occurs. During a supernova explosion, neutron capture processes responsible for the synthesis of radioactive elements take place under an abnormally high neutron flux. However, the origin of this large neutron flux still remains an open question. There are two related reactions:

$$n + \nu_e \to p + e^-, \qquad p + \bar{\nu}_e \to n + e^+$$

If $\bar{\nu}_e$ are more abundant than ν_e , then from above equations, neutron production is expected to dominate over proton production in the system. Hence, a possible conversion of ν_e to $\bar{\nu}_e$ due to gravity-induced oscillations could explain the observed overabundance of neutrons.

In the early universe, if oscillations occurred during an anisotropic phase, then at the GUT scale ($\tilde{B} \sim 10^5$ GeV), the corresponding oscillation length is found to be $L_{\text{osc}} \sim 10^{-24}$ km , according to the previous equations. This length is about 10^{14} times larger than the Planck length, highlighting the immense scale difference involved. Such a result is significant because, during the GUT era, the size of the universe was roughly 10^{26} times the Planck length. These conditions indicate that the oscillations could drive leptogenesis and subsequent baryogenesis via sphalerons, offering a possible origin for the universe's matter–antimatter asymmetry.

Summary :

In this work the authors investigate how the presence of a gravitational field (i.e. a curved spacetime background) can induce oscillations between neutrinos and antineutrinos (i.e. particle–antiparticle mixing).

We get this because of the extra B_a term which we get due to covariant derivative of spinor in curved spacetime. This term is multiplied to a term which is CPT odd in the lagrangian.

The term B_a need not be CPT odd in cases like kerr metric or inhomogeneous metric in general. Thus, we get a lagrangian which is not CPT even. This lagrangian gives rise to degeneracy breaking between neutrino and anti-neutrino giving rise to neutrino - anti neutrino oscillations.

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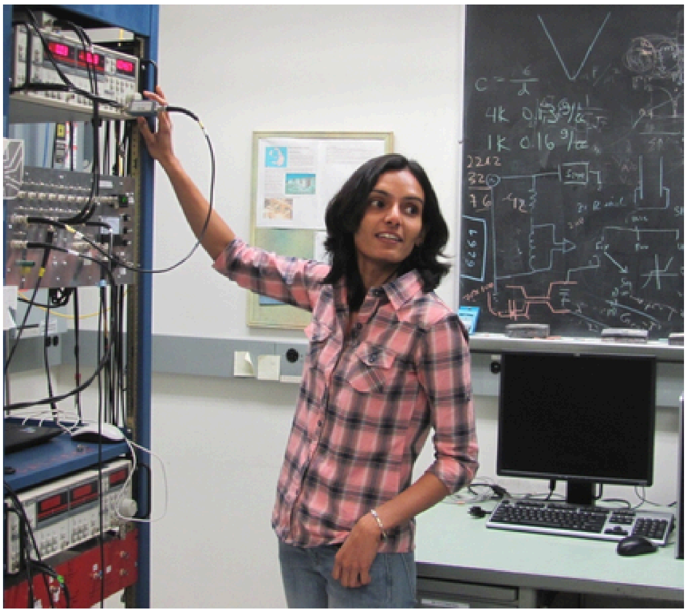
TWISTS AND LAYERS: EXPLORING THE LOWER DIMENSIONAL WORLD

Kalpesh Bhatnagar

It was a quiet afternoon when we met Professor Chandni in her office. The room was neatly arranged, with books and papers stacked in a manner that seemed to reflect the structured precision of her work. She carried the rare blend of warmth and authority that made us feel at ease within minutes.

Since joining the Department of Instrumentation and Applied Physics at IISc Bangalore in May 2017 as an Assistant Professor, Dr. Chandni U has built a research group focused on exploring the frontiers of low-dimensional materials and the emergent electronic phenomena, particularly in graphene.

As a former postdoctoral scholar at Caltech's Institute for Quantum Information and Matter, she brings a blend of global exposure and homegrown expertise to her work. Her journey in physics began in Kerala. Her initial interest was in learning about stars and planets. It was not until her master's at IISc, under the mentorship of Professor Arindam Ghosh, that she discovered her passion for condensed matter and experimental research.



Prof. Chandni Usha

Graphene and 2D Materials :

Professor Chandni has been working with Graphene for the past 10 years. It was one of the first 2D materials to capture global attention. Discovered in 2004, graphene's low resistivity, remarkable strength, and high flexibility quickly earned it a reputation as a 'wonder material.' By the early 2010s, Graphene was considered a contender to replace silicon. However, when we asked Professor about this hype and how the progress is so far, she pointed out a crucial limitation: *"Graphene is a semi-metal, not a semiconductor. That's why it could never really replace silicon. There's no band gap, so making semiconductor devices out of it is very hard."* While engineering a band gap has been attempted, the challenges have kept graphene from revolutionizing electronics in the way many first imagined. But graphene's limitations also opened the door to a broader family of 2D material**s**, many of which possess band gaps. "Materials like MoS_2 and WS_2 transition metal dichalcogenides are semiconductors and better suited for applications," she explained. These materials can be exfoliated down to monolayers, much like graphene, but can also be grown as single layers, making them suitable for device fabrication. "A lot of companies like Samsung and TSMC are investing heavily. The challenge is that it is difficult to grow them with high quality at large scales. To make an electronic device, you need to pack billions of transistors. That's still a bottleneck."

"When you rotate two layers slightly with respect to each other, that system can become a metal, an insulator, a superconductor, or even a magnet,"

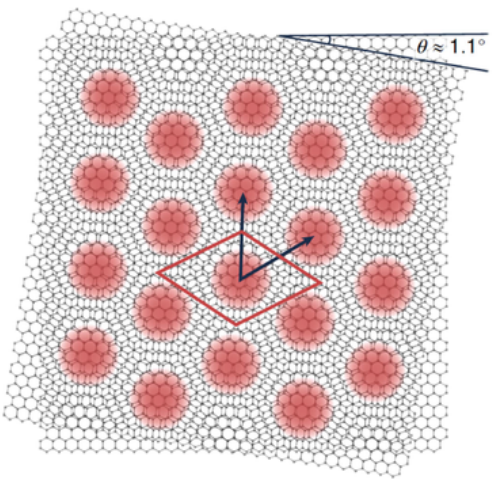
Moiré Graphene and the Magic Angle :

We moved on to one of the more exciting research topics in the field, Moiré graphene systems, on which Chandni's group has worked extensively. Around 2018, researchers discovered that stacking two graphene layers at a tiny relative angle produces entirely new properties. *"When you rotate two layers slightly with respect to each other, that system can become a metal, an insulator, a superconductor, or even a magnet," she explained. For condensed matter physicists, this meant a single material could host nearly every phase usually studied in separate systems.*

Chandni shifted her focus to this new material. "It was very easy for us; we were trying to align the layers, which is very difficult. So, it's obviously easy to just give it a small misalignment," she said. We asked her about the 'magic angle', a term that we frequently encountered while reading about twisted bilayer graphene. She explained how the angle between the layers affected the width of the energy bands of the system. "At about 1.1 degrees, the bands become really flat. The bandwidth is minimized. That's the 'magic angle.' The electrons then interact very strongly, and that's where you see all these unusual phases. Move slightly away, and the properties vanish.

We were curious how these several phases are attained in a single system. She clarified that these phases do not coexist. It is only upon changing certain parameters that we get the different phases. "You can get the phases by changing the concentration of electrons that you have in your band. You can also change the temperature, or the magnetic and electric fields."

When we asked her whether she ever considered pivoting and exploring new topics in this area as the field evolved, she answered pragmatically. "As an experimentalist, I can't simply jump fields. The equipment puts constraints. But we can refine the measurement techniques, work with new materials, or make changes to the fabrication process while retaining the core equipment and measurements. These are the directions we can go in."



Life at IISc :

Shifting the conversation away from research, we asked her about life at IISc. Professor Chandni has been both a student and a professor at IISc, and we were keen to get her unique perspective. *"When I was a student, there were no undergraduates. The total strength was about 2,000. In my class, there were only eight students. Today, the total strength has more than doubled; you see a lot more people around," she recalled.*

The second difference she mentioned was in the facilities. "Back then, doing an experiment was harder—you had to book slots far in advance, and things were a little slow. Today, the access is far easier."

Teaching Undergraduates :

From there, the discussion turned to her teaching experiences. She has taught a wide range of lab and theory courses, from electricity and magnetism lab for freshman undergraduates to specialized courses in solid state physics and quantum technologies. She reflected warmly on her interactions with undergraduate students. "The UGs are often the top students. They engage very well, and I feel I've learned a lot from them."

Still, she acknowledged that this enthusiasm can sometimes give rise to over commitment. Many undergraduates, she noted, take on multiple projects at once. "You look at a CV, and you've done like 100 different things, it doesn't really help, right?" she remarked, highlighting how research and training require time and focus of the students and the mentors alike. She pointed out how peer pressure drives this. "Students are all over the place with their projects. This happens often. Mostly it's peer pressure," she said. Summing up her experience with undergraduates in her lab, she said, "We've had fantastic UGs who did very well, and we've had some bad experiences too. It's a spectrum."

"Students are all over the place with their projects. This happens often. Mostly it's peer pressure,"

Advice to Young Researchers :

As our conversation drew to a close, we asked her what advice she would give to the undergraduates drawn towards condensed matter physics, particularly its experimental aspects. Her answer was striking in its honesty. *"Bright students often drift to theory because they're used to doing mathematical things. During school,* we don't have that much experimental exposure. *Experimental physics demands a different skill set and a different attitude."*

"Experimental physics demands different skills and a different attitude."

She recalled how an undergraduate student once questioned her. "How can you be spending your life doing this?" This is where she emphasizes the importance of persistence. "A hundred things will fail. It is boring sometimes, repetitive. But the moment you see a result, it's very fulfilling."

As we left, we carried with us the sense that her perspective was both pragmatic and deeply motivating. The study of low-dimensional systems may still be a young field, full of unanswered questions and technical challenges. Yet, in Professor Chandni's words, and in her example, we found a reminder that true discovery depends on more than intellect—it is fueled by patience, resilience, and a curiosity that endures setbacks.

"A hundred things will fail. It is boring sometimes, repetitive. But the moment you see a result, it's very fulfilling."

EMERGENT GRAVITY: A PARADIGM SHIFT

Sachin Ghongde

Research in the last 10-20 years suggests that the field equations governing gravity have the same conceptual status as the equations of elasticity and hydrodynamics in a wide class of theories, including but not limited to Einstein theory, for example the Lanczos-Lovelock models. This further suggests that the spacetime dynamics is just the emergent statistical mechanics of some underlying, unknown degrees of freedom and hence this new Emergent paradigm provides a completely thermodynamic interpretation of spacetime. I will try here to discuss how this comes about by reviewing the latest progress in this area of research.

“While there has been a lot of very interesting and imaginative work done .... it is safe to say that nothing which could definitely be called progress has been accomplished."

-- Lee Smolin, 1979

Introduction:

This qoute from Lee Smolin remains valid upto a large degree to this day. While there have been several theories with sophisticated mathematics like String Theory, Loop Quantum Gravity, Causal Dynamical Triangulations (CDT) etc, quantizing gravity to understand the quantum origin of spacetime and gravity still remains a job, far from finished.

Here, we'll start by looking at various points of conflicts and contacts in bringing together gravity and the quantum theory. Next, we will look at gravity from the perspective of a completely new paradigm of Emergent or Thermodynamic gravity. I'll be presenting the evidences of Gravity being thermodynamic in its origin and the implications of this perspective.

Points of Conflict and Contact between Gravity and Quantum Theory :

- **Non-Renormalizability of the metric tensor :** In QFT, a field is thought of as a system with infinite harmonic oscillators. A field with no interaction of any kind, a free-field is constructed out of uncoupled harmonic oscillators. Since there are infinite number of degrees of freedom involved, certain quantities like energy can diverge as well. However, this issue can be dodged by choosing to write oscillators in a particular representation, which are allowed in the commutator algebra of the theory. But the situation gets a little tricky once the the the perturbations are switched on. Then the perturbation series for certain quantities like scattering amplitudes or the propagator is in general not guaranteed to converge and has to be interpreted as an asymptotic expansion. Moreover, even the terms in the series may not be finite, which is related to the fact that the virtual quantas of arbitrarily high energies are allowed in the theory. This serious issue is resolved by the so-called Renormalization approach, where we redefine the coupling constants of the field like mass, charge in a particular way to cure the infinities of the perturbative series. This method however doesn't work when we treat the metric tensor $g_{\mu\nu}$
- as a tensor field variable as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa g_{\mu\nu}$, it turns out that it is not perturbatively renormalizable. There is no single redefinition of the fields that can renormalize the theory.
- **Interplay between spacetime and quantum fields:** The fields in QFT are described with a Minkowskian spacetime in the background. However, spacetime itself is a dynamical entity in General Relativity. Any quantum theory of gravity must therefore take this into account.
- **Information loss due to horizons:** Since gravity is the curvature of spacetime, only a limited region of spacetime is accessible to an observer, very much like we can't see past the seashore horizon due to the curvature of earth. This loss of information of a certain region of spacetime and therefore of the quantum fields in these regions is a phenomena not present in any other form of interaction. Therefore, this requires a reformulation of the equations governing quantum fields by tracing over the inaccessible degrees of freedom of the entire system.

- **The myth of a free-field:** Since any form of energy must curve spacetime, hence gravitates, this rules out the existence of non-interacting fields. Even in the absence of any external interaction, any non-trivial field configuration possess certain amount of energy and therefore it should gravitationally couple to itself. This makes all matter fields self-interacting. However, the formulation of QFT even in a given background curved geometry is itself challenging, non-trivial task, much less ask about any quantum description of the background spacetime.

The conventional quantum field theory as we know works all well when we have a static causal structure, flat background geometry / a global Lorentz frame. But gravity removes all these features right away. Considering all these points, there is a possibility that perhaps we might be looking at the situation in a fundamentally incorrect way.

Thermodynamics of Spacetime Horizons:

The assignment of a thermodynamic entropy to the event horizons of black holes was the first connection between spacetime geometry and its thermodynamics. Stephen Hawking proved that any classical process involving black hole cannot result in a process where the area of its horizons can't decrease. Later, Bekenstein suggested that the horizons of black holes must be attributed an entropy, proportional to its area.

Temperature of Horizons and the evidence for microstructure of spacetime : Standard Einstein's theory result of the black hole horizons having an entropy, implies the existence of their temperatures as well. However, it can be shown that any arbitrary spacetime with horizons possess a certain temperature with the help of standard QFT. Here, we will look at the simplest case of uniformly accelerated frame or Local Rindler Frames(LRF). It turns out that the vacuum fluctuations in the free falling frame around any event, appear to be a thermal state in the LRF. Let us consider the standard Minkowskian space(1+1) with the line element as

$$ds^2 = e^{2gx}(-dt^2 + dx^2) = -dT^2 + dX^2$$

for the LRF and inertial frame respectively. After a coordinate transformation, it becomes $ds^2 = dl^2 + g^2 l^2 dt^2$, where $l = g^{-1} e^{gx}$ and g being the acceleration. The null surfaces are mapped by the null rays following $X = \pm T$, bifurcates the chart in 2 halves L and R, which are causally disconnected, with ϕ_L and ϕ_R as the field configurations respectively. After a Wick rotation of the LRF time as $igt = gt_E$ and inertial time as $T_E = iT$, the line element becomes $ds^2 = g^2 l^2 dt_E^2 + dl^2$ = $dT_E^2 + dX^2$. Here, it follows that the evolution in T_E will take the field configuration from $T_E = 0$ to $T_E \to \infty$, while in LRF, the angular time coordinate t_E evolves from 0 to $2\pi g$.

Now, in the free falling frame, the global vacuum functional can be represented in terms of ϕ_L and ϕ_R :

$$\langle \mathrm{vac} \mid \phi_L, \phi_R \rangle \propto \int \mathcal{D}\phi \, e^{iS[\phi]}$$

$$= \int_{T_E=0}^{T_E=\infty} \mathcal{D}\phi \, e^{-S[\phi]}$$

$$= \int_{t_E=0}^{t_E=\pi} \mathcal{D}\phi \, e^{-S[\phi]}.$$

Using the Heisenberg picture, if we try to describe the system in terms of the position and momentum operators $\hat{q}$,$\hat{p}$, then the kernel, representing the probability amplitude for a particle to propagate from (t_1, q_1) to (t_2, q_2) can be represented as the matrix element $\langle q_2, t_2 | q_2, t_2 \rangle = \langle q_2, 0 | exp(t_2 - t_1) | 0, q_1 \rangle$, where H is the time independent Hamiltonian of the system. This allows us to show that; $\langle q_2, t_2 | q_2, t_2 \rangle =$

$$\langle q_2, 0 | exp(T) | 0, q_1 \rangle = \sum \psi_n(q_2) \psi_m^*(q_1) exp(-iE_n T),$$

where $\psi_n = \langle q_2 | E_n \rangle$ and $\psi_m = \langle q_1 | E_m \rangle$.

$$\therefore \langle vac | \phi_L, \phi_R \rangle \propto \int_{gt_E=0, \phi=\phi_R}^{gt_E=\pi, \phi=\phi_L} \mathcal{D}\phi e^{-S[\phi]} \propto \langle \phi_L | e^{-\pi H_R/g} | \phi_R \rangle$$

In order to describe the system in the region say R, we'll need to trace out the ϕ_L beyond the horizon, which gives us the thermal density matrix for the system in R as:

$$\rho(\phi_R^{'}, \phi_R) \propto \int \mathcal{D}\phi_L \langle \phi_L, \phi_R^{'} | vac \rangle \langle vac | \phi_L, \phi_R \rangle \$ \propto \langle \phi_R^{'} | e^{(-2\pi H_R/g)} | \phi_R \rangle$$

This corresponds to a horizon temperature $T = g/2\pi$. One very important thing to note here is that this result is completely independent of the field equations(if any) satisfied by the metric. So it is in this sense that the temperature of a spacetime horizon is a purely kinematic phenomena and has nothing to do with the dynamics of that spacetime.

Boltzmann's insight : The above result is way more general than the case of black hole horizons and extends beyond Rindler frame to all spacetimes equipped with horizons. It tells us that spacetime can get heated for certain class of observers and hence temperature becomes an observer dependent quantity. However, the very fact that you can heat up spacetime implies that it is made up of its own atoms or microscopic degrees of freedom. This should not come up as a surprise, as it was originally Boltzmann's insight that told us, if an object has the property to store some given energy internally in the form of heat, then there must be some internal degrees of freedom to hold the heat content. This suggests that spacetime also has some micro-structure.

Entropy unlike temperature depends on the field equations : Now the procedure for obtaining the entropy of the horizon as one would have thought would be to trace out the calculate $S = -Tr(\rho ln\rho)$ (usually called the entanglement entropy) with the help of the reduced density matrix of the field modes in region R. This turns out to be proportional to the area of horizon, however, it is divergent when spacetime is assumed purely continuous. This is handled by introducing minimum length cut-off(as some sort of lattice spacing) introduced by hand. Due to this, one fails to assign a unique value to the entropy using just QFT in an external geometry.

$$S \propto \mathcal{A}/L_p^2 \quad \text{(in spatial dimensions = 3);}$$

NOTE: Its often regularized in the literature, the use of an ultraviolet cutoff/lattice spacing to make sense of the situation. However, it lacks a clear physical justification, of course apart from the urge to get rid of meaningless divergenses in the theory. What this implies is the standard QFT which assumes a smooth flat background spacetime, is clearly a myth, just as a purely smooth continuous solid object is a myth.

Spacetime has a microstructure !

Now we shall see, that unlike temperature, the entropy of horizons is not independent of the field equations of gravity. Here, the gravity action functional plays a major role, as it contains information about the bulk and surface of a spacetime region.

$$\mathcal{A} = \int d^D x \sqrt{-g}[L(R^{ab}_{cd}, g^{ab}) + L_{matt}(g^{ab}, \phi_A)];$$

where L_{matt} is the matter Lagrangian (for some matter variables denoted symbolically as ϕ_A.) If this action is varied with respect to the metric with some appropriate boundary conditions, we get the following field equations:

$$G_{ab} = P_a^{\ cde} R_{bcde} - 2\nabla^c \nabla^d P_{acdb} - \frac{1}{2} L g_{ab} = R_{ab} - \frac{1}{2} L g_{ab} = \frac{1}{2} T_{ab};$$

where $\{P_{abcd} = \partial L/\partial R_{abcd}\}$. For the field equations to be second order in metric, L is chosen such that $\nabla_a P^{abcd} = 0$. A whole set of such L's which are constructed as polynomials of scalar curvature tensors comes from the so-called Lanczos - Lovelock models. The general Lanczos-Lovelock Lagrangian has the form:

$$L = \sum_{m=1}^{K} c_m L_m; \quad L_m = \frac{2^{-m}}{16\pi} R^{b_1 b_2}_{a_1 a_2} \cdots R^{b_{2m-1} b_{2m}}_{a_{2m-1} a_{2m}},$$

where the c_m are arbitrary constants and L(m) is the m-th order Lanczos-Lovelock Lagrangian. The m = 1 term is proportional to $\delta^{ab}_{cd} R^{cd}_{ab} \propto R$ and leads to Einstein's theory. The m = 2 term gives rise to what is known as Gauss-Bonnet theory. Because of the determinant tensor, it is obvious that in any given D we can only have K terms where $D \geq 2K$. It follows that, if D = 4, then only the K = 1,2 are non-zero. Of these, the Gauss-Bonnet term corresponding to K = 2 gives, on variation of the action, a vanishing bulk contribution. In dimensions D = 5 to 8, one can have both the Einstein-Hilbert term and the Gauss-Bonnet term etc and so on for higher dimensions. It is conventional to take $c_1 = 1$ so that the $L_{(1)}$, which gives Einstein gravity, reduces to ($R/16\pi$).

In Lanczos-Lovelock models, the field equations reduces to:

$$P^{de}_{ac} R^{bc}_{de} - \frac{1}{2} L \delta^b_a = \mathcal{R}^b_a - \frac{1}{2m} \mathcal{R} \delta^b_a = \frac{1}{2} T^b_a;$$

$$\mathcal{R}^b_a \equiv P^{de}_{ac} R^{bc}_{de}; \quad \mathcal{R} = R^a_a$$

In the simplest context of m = 1 we take $L = R/16\pi$ (with conventional normalization), leading to $P^{ab}_{cd} = \frac{1}{32\pi}(\delta^a_c \delta^b_d - \delta^a_d \delta^b_c)$ as well as $\mathcal{R}^a_b = \frac{R^a_b}{16\pi}, \quad \mathcal{G}^a_b = \frac{G^a_b}{16\pi}$ and one recovers Einstein's equations. The structure of the theory is essentially determined by the tensor P_{abcd} which has the algebraic symmetries of curvature tensor and is divergence-free in all indices.

There exists a Noether current J^a, corresponding to the infinitesimal coordinate transformation:

$x^a \rightarrow x^a + q^a$:

$$J^a = 2G^a_b q^b + Lq^a + \delta q v^a = 2R^a_b q^b + \delta_q v^a; \quad \nabla_a J^a = 0;$$

where $\delta_q v^a = (-g)^{-\frac{1}{2}}(q^A \pi^a_A)$ is the boundary term in the action which arises for the variation of the metric in the form $\delta g^{ab} = \nabla^a q^b + \nabla^b q^a$

Given that $\nabla_a J^a = 0$, we can introduce an anti-symmetric tensor J_{ab} by $J^a = \nabla_b J^{ab}$. For the Lanczos-Lovelock models, one can determine $\delta_q v^a$ and show that the J^{ab} and J^a can be expressed in the form:

$$J^{ab} = 2P^{abcd} \nabla_c q_d; \quad J^a = 2P^{abcd} \nabla_b \nabla_c q_d J^{ab}$$

Now, the Wald entropy of a blackhole horizon in an asymptotically flat spacetime can be written as the following:

$$S = \beta \int d^{D-1}\Sigma_a \, J^a = \beta \int d^{D-2}\Sigma_{ab} \, J^{ab}$$
$$= \frac{1}{4}\int_{\mathcal{H}} (32\pi P^{ab}{}_{cd})\, \epsilon_{ab}\epsilon^{cd}\, d\sigma$$

where Σ_a is the vector normal to the surface, Σ_{ab} , ϵ_{ab} are the bi-vector normal and $\beta^{-1} = \kappa/2\pi$ is the horizon temperature. J^a is the Noether current for the time translational vector (killing vector) $q^a = \xi^a$ for the asymptotically static black hole solution. In the final expression the integral is over any surface with (D − 2) dimension which is a spacelike cross section of the Killing horizon on which the norm of ξ^a vanishes. Thus horizon entropy is given by an integral over the horizon surface of the P_{abcd}.
(It must be noted that horizon entropy is proportional to its area only in Einstein's theory and is not a general result.)

Field Equations reduces to Thermodynamic Identities on Horizons : Its not only an algebraic accident that the field equations in a large class of theories including Lanczos-Lovelock models, three dimensional BTZ black hole horizons, FRW cosmological models in various gravity theories, Horava-Lifshitz gravity etc, when evaluated at the horizon of a static solution can be expressed in the form of the thermodynamic identity: TdS = dE + PdV, where S is the Wald entropy of the horizon in the theory, E is a geometric integral of the scalar curvature of the sub-manifold of the horizon and PdV represents the work function of the matter source. The differentials dS,dE etc should be thought of difference in S,Eg etc between two solutions in which the location of the horizon is infinitesimally displaced. For example, in the simplest context of spherically symmetric horizon in Einstein's theory [with $-g_{00} = g_{11}^{-1} = f(r)$ with f(a) = 0 determining the location of the horizon at r = a], the field equation on the horizon reduces to:

$$\frac{c^4}{G}[\frac{\kappa a}{c^2} - \frac{1}{2}] = 4\pi P a^2$$

where $\kappa = f'(a)/2$ is the surface gravity and P is the pressure of the source. Now if we multiply both sides of this equation by da, it could be expressed in the form:

$$\underbrace{\frac{\hbar\kappa}{2\pi c}}_{k_b T}\underbrace{\frac{c^3}{G\hbar} d\left(\frac{4\pi a^2}{4}\right)}_{k_b^{-1} T} \underbrace{- \frac{c^4 da}{2G}}_{-dE} = \underbrace{P d\left(\frac{4\pi a^3}{3}\right)}_{PdV}$$

It must be stressed again, that this accident or one might say miracle happens in all gravitational theories and is much more general than Einstein's theory. On top of that, having evaluated the temperature of a horizon using the QFT method, we can read off its entropy as well, by just calculating the field equations of the theory on the horizon.

Holographic Duplication and Action as the Free Energy of Spacetime : We now sort of have this rough idea that all the dynamics of the theory must be contained in the action functional. Indeed, we can realize this even further by looking at the structure of the action. In almost all gravity theories, the action has a bulk term and a surface term. After ignoring the surface term or canceling it with a counter term and setting the bulk term equal to zero, we obtain the field equations. However, it turns out that if we evaluate the surface term on the horizon of a spacetime, we obtain exactly the entropy of it. Now, how does surface term which we discard, know anything about the entropy coming from the field equations of the theory. This isn't just another algebraic accident, instead one can actually show that the bulk and surface terms are actually related as:

$$\sqrt{-g}\,\mathcal{L}_{\rm sur} = -\partial_a(g_{ij}\,\frac{\delta(\sqrt{-g}\,\mathcal{L}_{\rm bulk})}{\delta(\partial_a g_{ij})})$$

All Lanczos-Lovelock models holds this relation. Infact, if we demand the action to be function of the curvature tensor and still have 2nd order field equations, the above relation is essential. This duplication of information between the bulk and surface region shows us that all the dynamical content of a spacetime region is actually stored in its boundary rather in the bulk. Yet another crucial and straightforward realization is again contained in the form of the action. The fact that action has bulk and a surface term plus surface term gives us teh entropy yields action to have the form of a sort of free energy of spacetime. This can be easily seen for any Lanczos-Lovelock model by writing only the time component of the Noether current for the time translation vector (killing vector), $q^a = (1,0)$ in the following form:

$$L = \frac{1}{\sqrt{-g}}\,\partial_\alpha\left(\sqrt{-g}\,J^{0\alpha}\right) - 2\mathcal{G}^0_0$$

Here, only the spatial derivatives contribute in a static geometry. After integrating $L\sqrt{-g}$, the first term gives the entropy and the second term energy, making the action the free energy of spacetime.

The Atoms of Spacetime and their Thermodynamics:

Equipartition law and Wald entropy from it : After Boltzmann's conjecture of different gases made of discrete atoms, it was realized that they store some energy E via some n internal degrees of freedom in the form $E = 0.5 k_b T$ i.e. each degree of freedom stores energy $0.5 k_b T$. This is the law of equipartition. It is remarkable that in our thermodynamic approach to gravity, there exist an equipartition law as well.

$$E = \frac{1}{2k_B}\int_{\partial V}\frac{\sqrt{\sigma}\,d^2x}{L_p^2}\frac{Na^\mu n_\mu}{2\pi} \equiv \frac{1}{2}K_B\int_{\partial V} dn\, T_{\rm loc}$$

(where $T_{loc} = (Na^\mu n_\mu)/(2\pi)$ is the local acceleration temperature and $\Delta n = \sqrt{\sigma} d^2x/L_p^2$ thereby allowing us to read off the number density of microscopic degrees of freedom. The true elegance of this result again rests on the fact that it holds true for all Lanczos-Lovelock models. For a Lanczos-Lovelock model with an entropy tensor P^{ab}_{cd} one gets the result;

$$E = \frac{1}{2k_B}\int_{\partial V} dn\, T_{\rm loc}$$

$$\frac{dn}{dA} = \frac{dn}{\sqrt{\sigma}\,d^{D-2}x} = 32\pi P^{ab}_{cd}\,\epsilon_{ab}\epsilon^{cd}$$

where ϵ_{ab} is the binormal on the codimension-2 cross-section.

Since the density of microscopic degrees of freedom obtained here suggests that the entropy associated with a general surface in Lanczos-Lovelock models (or the entropy associated with a horizon in a more general theory) will be proportional to an integral over $P^{ab}_{cd}\,\epsilon_{ab}\epsilon^{cd}$.

$$S \propto \int_{\partial V} dn \propto \int_{\partial V} 32\pi P^{ab}{}_{cd}\,\epsilon_{ab}\epsilon^{cd}\,\sqrt{\sigma}\,d^{D-2}x$$

This is exactly the expression for Wald entropy, but its remarkable that we have obtained it only from the equipartition law!

Gravitational field equations from a Thermodynamic Extremum Principle : Just like we can obtain the effect of gravity on matter can with the help of the "equivalence principle", "special relativistic behavior in all locally inertial frames", we can now also obtain the effect of matter on spacetime using the "thermodynamic extremum principle", "a suitable thermodynamic potential of the microscopic degrees of freedom should be extremised for all local Rindler frames". To every null vector $n^a(x)$, we associate a thermodynamic potential $\mathcal{I}[n^a]$, which is quadratic in n^a :

$$\mathcal{I}[n^a] = \mathcal{I}_{\rm grav}[n^a] + \mathcal{I}_{\rm matt}[n^a]$$
$$\equiv -4P^{cd}_{ab}\nabla_c n^a \nabla_d n^b - T_{ab}n^a n^b$$

where P^{ab}_{cd} is a tensor having the symmetries of curvature tensor and is divergence-free in all its indices and T_{ab} is a divergence-free symmetric tensor. We now demand that $\frac{\delta \mathcal{I}}{\delta n^a} = 0$ for the variation of all null vectors n^a with the condition $n^a n_a = 0$, imposed by adding a Lagrange multiplier function $\lambda(x) g_{ab} n^a n^b$ to $\mathcal{I}[n^a]$. Now;

$$\frac{\partial \mathcal{I}}{\partial(\nabla_c n^a)} = -8P^{cd}_{ab}\nabla_d n^b;\ \frac{\partial \mathcal{I}}{\partial n^a} = 2\,[T_{ab} + \lambda(x) g_{ab}]n^b$$

the Euler-Langrange equations reduce to:

$$\nabla_c[-8P^{cd}_{ab}\nabla_d n^b] = 2\,[T_{ab} + \lambda(x) g_{ab}]n^b$$

Because of the condition $\nabla_c P^{cd}_{ab} = 0$ and the antisymmetry property $P^{cd}_{ab} = -P^{dc}_{ab}$ we find that all the derivatives disappear on the left hand side and we get:

$$(2\mathcal{R}^a_b - T^a_b - \lambda \delta^a_b)n^b = 0$$

where $\mathcal{R}^a_b = P^{jk}_{bi} R^{ai}_{jk}$. Since we want this condition to hold for an arbitrary null vector n^a, plus, using the generalized Bianchi identity and $\nabla_a T^a_b = 0$, we finally get:

$$\mathcal{G}^a_b = \mathcal{R}^a_b - \frac{1}{2}\delta^a_b L = \frac{1}{2}T^a_b + \Lambda\delta^a_b$$

where Λ is a constant. These are precisely the field equations that we get with a Lanczos-Lovelock lagrangian L (note that the cosmological constant Λ arises as an integration constant here.)

Concluding Comments :

As said earlier, I tried to present some of the evidences that points towards a thermodynamic description of gravity and also to a discrete microstructure of spacetime. I shall now summarize them:

- Spacetime like any other piece of matter can be attributed a certain temperature. Local Rindler frames sees thermal fluctuations of field modes, what the free-falling inertial frames sees as vacuum fluctuations. And so temperature is an observer dependent quantity.

- The entropy of a spacetime region, unlike temperature can't be assigned a unique value via standard QFT (due to the fact that the standard QFT assumes spacetime as a perfectly smooth continuum at all scales, which its not!). And its entropy and not temperature is what contains the dynamical content, very much like a glass of milk and a steel, both could be at the same temperature, in spite of having completely different internal structures.

- Gravitational field equations precisely reduce to standard thermodynamic identity, when evaluated on spacetime horizons. This not only gives gravity a thermodynamic status, but it also enables us to read off entropy in a wide class of theories.
- Again, the thermodynamic origin of gravity is advocated by the existence of surface and bulk terms in the action functional. Action can be interpreted as the free energy of spacetime. On top of that there seems to be holographic duplication of information of region inside horizons on the horizon boundaries.
- There exists an equipartition law, just like for ideal gases, implying strongly that spacetime has got a microstructure and the degrees of freedom are at the scale of Plank's area.
- It is possible to obtain the field equations of the theory, by extremising a suitable thermodynamic potential (entropy, free energy, enthaply etc) in a given spacetime boundary. In this approach, the cosmological constant arises just an integration constant.

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THE QUANTUM WORLD OF DR. SUBROTO: A DEEP DIVE INTO CONDENSED MATTER PHYSICS

Amrita Notani, Kabyadeep Das

In this interview, Dr. Subroto, a renowned condensed matter theorist, shares insights into his research, and his journey into physics, and offers valuable advice for aspiring physicists. From strongly correlated systems to Berry phase effects, Dr. Subroto discusses the fascinating world of quantum mechanics and condensed matter physics with remarkable clarity and depth.

Prof. Subroto Mukerjee

Let's start with your work. What exactly do you do as a condensed matter physicist?

I am a condensed matter theorist. I work broadly in the area of what people often call strongly correlated systems, which are condensed matter systems where quantum concepts are important, and the constituent elements interact strongly with one another. For instance, if you look at an ordinary metal like aluminium or copper and are interested in its electronic properties, like electrical or thermal conductivity, you know that conduction happens due to the flow of electrons, which are charged particles and thus interact via the Coulomb interaction. However, for conventional metals, this interaction can be largely neglected, thanks to the fermionic nature of electrons and the nature of the Coulomb interaction. We can essentially treat these electrons as free particles, which is quite powerful, it explains the interesting properties of most conventional metals and can be extended to include slightly unconventional situations like superconductivity.

But over the last four decades, we've encountered systems where this approach doesn't work anymore. The interactions between electrons are so strong that they cannot be neglected. And unlike conventional metals, where we have an overarching paradigm that lets us ignore these interactions, for these strongly correlated systems, no such broadly applicable paradigm exists. That's one of the holy grails in our field, to develop such a paradigm. But no one has been able to do it so far, and there might be good reasons why it may not even be possible. So what we try to do instead is look at classes of materials with similar phenomena and develop theories to explain those specific phenomena in that class of materials. I should mention that the name "condensed matter physics" suggests systems where there's some condensed collection of constituents, but these don't necessarily have to be electrons. There are experimental systems, like cold atoms that condense into a fluid or liquid, where similar ideas apply, and the interactions might be strong. Depending on the type of atom, the elementary constituents can be either fermions or bosons, which adds another dimension of complexity. Broadly speaking, I try to understand the physics of these systems and their various properties. More specifically, I'm interested in transport in these systems, electrical transport, thermal transport, because transport coefficients like electrical conductivity or thermal conductivity can tell us a lot about the underlying physics of the system. I'm also interested in how statistical mechanics emerges in these quantum systems. When you're looking at large collections of particles say, 10^{23} particles, it's impossible to describe the system in terms of individual particles. You need a statistical description, which is fine because the properties we typically measure are aggregate properties, not properties of individual particles. In classical systems, we have a good understanding of how statistical mechanics emerges from the underlying classical laws. For quantum systems, there's also some understanding, but it's still a more nascent field with many unresolved aspects.

One particularly interesting phenomenon in quantum systems is that, unlike classical systems which eventually reach thermal equilibrium, some quantum systems genuinely don't reach equilibrium. There's at least one class of quantum systems that don't thermalize at long times, meaning the normal rules of statistical mechanics don't apply to their long-time state. What replaces these notions is still not fully understood, and I'm interested in studying these systems and the transitions between thermalizing and non-thermalizing behavior.

How did you get into the field of condensed matter physics? And how accessible is it for newcomers?

My journey into condensed matter physics was more of a random walk than a well-thought-out path. I didn't particularly gravitate toward condensed matter physics as an undergraduate. After my bachelor's in physics, I came to IISc as an integrated PhD student, mainly attracted by the institution's reputation rather than specific research areas. At that time, the physics department here was heavily focused on condensed matter physics, so I naturally ended up in that field. I actually started as an experimentalist, doing optical measurements on semiconductors for my master's. Then I went to Princeton for my PhD, initially intending to continue experimental work. While there, I realized two things: I wasn't particularly skilled at experiments, and I was increasingly drawn to theoretical physics after taking courses taught by renowned theorists. Since I had already spent a year in graduate school, I decided to switch to theory but stay within condensed matter physics since I already had some familiarity with it. So that's how I became a condensed matter theorist through a series of somewhat random decisions rather than a deliberate plan. Now, regarding how someone can enter this field if they're already interested: Different people will give different answers, but based on my experience and observations, I think the most important thing is first to develop a very strong foundation in the basics. Even though your ultimate goal might be research in condensed matter physics, at the undergraduate level, it's more important to focus on developing a strong background in quantum mechanics, statistical mechanics, classical mechanics, electromagnetic theory, and the necessary mathematical skills, without worrying too much about how these will be applied specifically to condensed matter research. You might wonder if this advice is too general, wouldn't it apply equally to high-energy physics or astrophysics? On one level, yes. At your stage, becoming strong in the basics is more important than specialization. But with condensed matter physics, perhaps more than other branches, there's less specialization within the field itself. While astrophysics has clear compartmentalization (cosmology, galaxy formation, astrophysical plasmas), in condensed matter physics, even with the broad division between hard and soft condensed matter, the principles used across sub-branches are often common. So I would advise undergraduate students to focus on strengthening their basics through their third year. Then, start exploring condensed matter specifically, learning about different systems and recognizing the underlying connections. This preparation will serve you well when you undertake a fourth-year project, where you can work on a specific area and determine whether you prefer theoretical or experimental work.

“So I would advise undergraduate students to focus on strengthening their basics through their third year. Then, start exploring condensed matter specifically, learning about different systems and recognizing the underlying connections.”

Is it manageable to balance teaching and research as a faculty member?

It is manageable because it has to be. In the US, for example, faculty members earn their salary primarily through teaching, not research. That's why during the three summer months when there's no teaching, American universities don't pay their faculty a salary they have to use their research grants to pay themselves during that time. So teaching is a very important component of your professional responsibility, along with research. The real question becomes how you divide your time between them. And there's also a third component people don't talk about much: administration. That involves serving on various committees like student affairs, faculty hiring, departmental management, fund allocation, and so on.

So someone like me and my colleagues have to balance research, teaching, and administrative work. Most people would probably say they want to spend at least 50% of their time on research and divide the rest between teaching and administration. But in my case, over the last few years, I've spent less than 50% on research, which I'm not happy about. So in that sense, teaching and administration are getting in the way of research, but that's something I've allowed to happen. It doesn't have to be the norm, and I plan to change that. Between teaching and administration, I'd much rather do less administration than less teaching. Teaching at least offers some intellectual reward. Administration, at least in my experience, doesn't. But it is part of your responsibility and the service you're expected to perform. Many of my colleagues in the department have done a much better job of maintaining that balance between research and other responsibilities. I have not.

You've received teaching awards over the years. How has being a good teacher affected your research?

Some people would argue that getting a teaching award is actually bad for your research because it suggests you're spending too much time teaching, time that could have been spent on research. There's an interesting anecdote about a young assistant professor at Cornell who was up for tenure and had won a teaching award. When the tenured faculty were discussing his promotion, several raised objections, saying, "He shouldn't be given tenure, he's got a teaching award, which shows he's spending too much time on teaching, time that could have been spent on research." But in my case, awards aside, teaching courses, especially teaching different courses, has definitely helped my research. As I mentioned earlier, having a good grasp of the basics is crucial for condensed matter physics research. Even if you've developed that foundation as a student, if you don't stay in touch with those fundamentals, some details fade over time.

“When you are doing research, you don't have brilliant ideas every day. Even for people like Feynman, let alone people like me, most days involve a lot of drudgery and frustration. You hit walls. No new ideas come.”

Teaching helps recover all of that knowledge because you have to revisit those details for your courses. And often, those details turn out to be important for your research as well.

Let me give you a recent example. Some of my recent research has involved looking at transport in condensed matter systems, specifically examining Berry phase effects. Berry phases are a fundamental concept in quantum mechanics related to the geometrical aspects of quantum evolution. They also manifest in condensed matter systems, producing specific physical effects. This wasn't something I had explored in my earlier research. But for the last couple of years, I've been teaching Quantum Mechanics II, which covers the Berry phase. I had learned about Berry phases as a student and found them intriguing, but I hadn't had occasion to apply them in my research. Many technical details had faded from memory.

When I revisited the Berry phase for teaching purposes, all those details came back, and that ultimately steered me toward this new line of research. So perhaps if I hadn't taught Quantum Mechanics II, I might not have worked on that particular aspect of transport, even though it's been a known phenomenon for quite some time. There's another, more abstract benefit to teaching that I've come to appreciate. Richard Feynman once talked about his contemporaries who worked at pure research institutes rather than universities. He was somewhat dismissive of them, arguing that their research productivity and quality actually declined once they were removed from teaching environments. His reasoning was that when you're doing research, you don't have brilliant ideas every day. Even for people like Feynman, let alone people like me, most days involve a lot of drudgery and frustration. You hit walls. No new ideas come.

That kind of experience can lead to intellectual stagnation without external stimulation. If you're teaching, and you hit a wall in your research, something completely unrelated to your course material might provide a fresh perspective. You might have to relearn something just for teaching purposes, and that can stimulate you intellectually and revive your creativity. So that's an indirect way teaching benefits research, it keeps your mind active and engaged with a variety of concepts, which can ultimately enhance your creative thinking for research.

“Doing cutting-edge research in any of these fields is increasingly difficult. It's very challenging to make impactful contributions in any particular field without having worked in it for several years.”

Should researchers focus on a specific area or explore multiple fields?

That's an excellent question, and the answer depends on who you ask and when you ask them. The research landscape has evolved significantly over time. Today, I would advise not spreading yourself too thin. It's important to have a well-rounded education in physics, with exposure to all the foundational areas like classical mechanics, quantum mechanics, statistical mechanics etc., and to explore various sub-areas. But when it comes to research itself, having developed that grounding, it's probably wise to maintain some focus.

In contemporary physics, we have these broad areas: condensed matter physics, high energy physics, and astrophysics. Doing cutting-edge research in any of these fields is increasingly difficult. Physics is an old subject with a rich history, and new experimental data is constantly coming in, fueling growth within each field. It's very challenging to make impactful contributions in any particular field without having worked in it for several years.

Let's say you're a condensed matter theorist who becomes fascinated by astrophysics after attending a talk. You might think, "This is exciting I want to work on this. I have a good foundation in physics; I should be able to make the switch." But the reality is, even with that strong foundation, if you haven't been working in that field, you'll likely enter at a very basic level. It will take considerable time to reach a point where you can make cutting-edge contributions. So jumping from condensed matter physics to astrophysics or biology-related physics, expecting to make meaningful contributions in all these areas, is extremely optimistic and practically impossible. Very few people manage to do that. Even within condensed matter physics, there's a significant division between experiment and theory. The techniques are so different that it's nearly impossible for someone to make substantial contributions in both areas.

That said, within theory, you have subfields like hard matter and soft matter, and within hard matter, topics like transport, quantum phase transitions, and others. With enough experience and maturity, you can work across these different subfields. Many of us in the department have done that. But that's not how we started. During our PhDs or postdocs, we were much more focused. With accumulated experience, and often by collaborating with people already established in a different subfield, we gradually branched out. That's the best way to transition: collaborate with someone who's already there, who can guide you through the process. This evolution takes time, so it's really only at a certain career stage that you can or should attempt this kind of diversification. Initially, given the current state of research, it's probably best to maintain focus. I wouldn't have given this same answer when I was starting my PhD. Back then, people had broader interests, and their dissertations were more wide-ranging than mine. By the time I began my doctoral work, research was already becoming more specialized. Now, it's even more so.

How do you decide what problem to focus on in your research?

In my case, my trajectory, whether in choosing a broad field or a more specific area, was largely driven by selecting a particular institution or person to work with. But I'm not suggesting that everyone should make decisions this way, that what you ultimately study should be determined by wanting to work with someone just because they were a good teacher or something similar. I think a better approach is this: once you've moved beyond the stage of strengthening your basic knowledge and are considering which branch of physics to pursue for your PhD, start exposing yourself to current research. Attend talks. Try reading research papers, not the highly technical ones at first, as you won't understand them. There are different types of papers, ranging from very technical to more pedagogical reviews. Start with accessible articles like those in Physics Today, then gradually move to more technical but still pedagogical papers. This will give you insight into important current research topics and the techniques they involve.

For instance, certain topics might require extensive computation. If you find that you prefer computational work over analytical approaches, or vice versa, you'll naturally gravitate toward certain areas. But to develop these preferences, you need to understand what techniques are involved in different fields. For that, you need to read, attend presentations, and talk with researchers. I'd also recommend talking to faculty members in your physics department about their research. Although finding the time can be challenging, both for you and for them. One approach for undergraduates might be to occasionally invite faculty members to give brief presentations about their research at a level you can understand. This would help you grasp what's happening in the field, who's working on what, and what methods they're using.

How can an undergraduate with limited experience work in condensed matter physics or contribute to a research project?

The undergraduate curriculum here requires students to do research through fourth-year projects, and for those continuing to a master's, fifth-year projects as well. Many undergraduates also pursue summer research projects, and quite a few have gotten their names on published papers. So clearly, it's possible for undergraduates to make meaningful contributions to research, even without extensive prior exposure to cutting-edge techniques.

But how? What typically happens is that undergraduate projects are designed to require very specific, well-defined tasks. You might need to write code to diagonalize a certain matrix, solve a particular differential equation, run a specific simulation, or perform some analytical calculation, like solving a set of differential equations or computing terms in a power series. You can approach these problems as purely mathematical challenges, without necessarily understanding the broader context. Whatever mathematical skills you've developed and basic physics you've learned in your courses is usually sufficient for these tasks.

You're not initially expected to propose extensions or interpret the results, you're given something specific to compute, and you do it. Though limited in scope, your work is still an important component of the larger project. This process teaches you two valuable things: To focus persistently on a particular problem until it's solved To move beyond the mindset of solving problems just for homework or exams. In coursework, if you solve a problem perfectly, you get an A+. If not, you might get an A or a B. But in research, there's no partial credit. You have to produce something that works, get a result, and that often involves trying multiple approaches, revising your work, and learning new techniques along the way. This represents an important shift in thinking. Additionally, while working on such projects, you're interacting with others involved, especially your advisor, who understands the broader context. They've formulated the problem and know what it contributes to.

Through these interactions, you begin to grasp the bigger picture, how your specific task connects to the overall research question. This exposure helps you understand how particular calculations or simulations relate to larger conceptual goals, and you start appreciating that skill. Eventually, as an independent researcher, you'll need to identify unsolved problems yourself, questions that connect to what others are working on and have broader interest.

LOOKING BEYOND THE VISIBLE UNIVERSE

Gunda Sai Vinay, Sachin Ghonde,
Chinmay Panchariya Anil

Prof. Nirmal Raj, from the Centre for High Energy Physics at IISc Bengaluru, works at the intersection of particle physics, astrophysics, and cosmology, focusing on understanding dark matter and hidden sector physics. His research explores theoretical models for dark matter and develops strategies to detect them through experiments, astrophysical observations, and extreme environments like neutron stars. By combining theory with innovative detection methods, his work aims to expand the ways we can probe the unseen components of the universe.

Prof. Nirmal Raj

Let's begin the interview. Professor, can you start by giving a brief of what exactly you do?

So I'm, by and large, theoretical particle physicist. Particle physics in general is the exploration of newer fundamental principles in physics, trying to understand nature at the most fundamental and microscopic level. There are many puzzles to be solved in particle physics. So we have right now, a model/theory in terms of quantum mechanics and relativity put together to describe most phenomena in nature, called the standard model. Most people in our department, CHEP, our work is to try to solve puzzles that show up in the standard model and beyond.

So different people are attracted towards different kinds of puzzles. The kinds of puzzle that I am most interested and in which I work on is dark matter. So in that sense I am more into astro particle physics, but you know, the astro part comes only because dark matter itself exists all over the universe but I really approach it from the particle physics point of view. The astrophysics is a necessary thing that I have to learn, but my work is mostly in particle physics.

So dark matter is basically an invisible substance that seems to make up most of the mass of the universe. I don’t know if you guys have heard about it. You'd have seen it in some YouTube video. So yeah, it makes up five sixth of the mass universe. It is dark. It does not emit, absorb or reflect light. Then you might ask, how do we know it's there? Well, through some affects of gravity. We don't need to actually see things to weigh them. So, we know it's there. We have many kinds of evidence for it. But a particle physicist then ask what does it mean? Right as a particle physicist, I know what the table is made of, I know what stars are made of. I'm not just describing the particles that are making them, I understand how the particles interact, what forces go into them and etc. in terms of quantum field theory. But I have no such idea for dark matter. I just know it's there. The mass is there. Does it even form structure or not? None of this is known. That is where my research process is and I try to approach it from many different angles.

I was in like one of your lectures, I think it's about "Six ways in which Dark Matter can kill you"? And the thing is, the only effects of dark matter we saw, as far as I know, are gravitational effects and Gravity is caused by the curvature of space time. But how do we know that only mass is causing by curvature ?

So in English, you know, when you just say gravity and all that, then it's not clear how would you know it's actually the mass? It's more subtle than that. It's the exact gravitation effect we have to see. Let's just go into the, a little bit of details. The fact that the, the galaxies are spinning faster than you expect, you can know there is more gravity. So you must have a lot of gravitation lensing, right? So we see these large structures where light bends around from far away in so many different ways. When you put together, you know that there are large invisible, clusters of mass there. There cannot be any other form of energy. It has to be a mass energy for you to get the right kind of light bending. Then there are more sophisticated observations, such as studying what is known as the cosmic microwave background. Simply put, this refers to photons that are remnants of the early universe. By carefully analyzing their frequency and intensity spectra, scientists can learn a great deal about the composition of the universe, specifically, how much of its total energy budget is made up of matter, radiation, and the mysterious component known as dark energy. Essentially, every form of energy contributes to the curvature of spacetime in some way. The precise effects of these contributions always trace back to the idea that there is an underlying gravitational "weight".

Why are people so confident that dark matter should be made up of some particle?

People are not. I'm not. Any serious dark matter physicist should not be so sure of it. We study dark matter as a particle since all known mass in the universe comes from particles. However, there are other possibilities as well. Dark matter could consist of black holes, not the stellar ones we observe today, since those are clearly ruled out by experimental constraints and other observations. Instead, it could be made of black holes that formed in the very early universe. These primordial black holes, much smaller in mass than asteroids, remain a possible candidate worth considering and part of my research program is to look for signatures of primordial black holes.

Yet another possibility, distinct from both black holes and particles, is that dark matter could arise from defects in the fields that fill spacetime. These are known as topological defects. Fields that carry energy can fill all of space, but under certain conditions, they can form stable configurations, localized distortions or knots in these fields. Such configurations could, in principle, contribute to or even make up the dark matter in the universe.

But are we not, in this context, referring to gravitons? If so, that would still imply a description of spacetime in terms of particles and quantized entities. In other words, even when we attempt to move beyond matter-based explanations, we ultimately return to a framework that expresses the structure of space through particle-like concepts.

That is a different idea. Everything we currently understand about gravity is based on the classical theory, which describes spacetime as a continuous and rigid geometric structure. From that perspective, one might then ask what the excitations of the gravitational field are. The gravitational field itself is represented by the metric tensor, and its quantum excitations would correspond to gravitons. However, this concept is distinct from that of topological defects, if that is what you are referring to. That phenomenon does not occur within the framework of topological theories. Topological theories primarily involve fields that exist within spacetime, such as scalar fields or similar entities, which can assume specific configurations, often leading to interesting physical implications. However, spacetime itself is not one of these fields; it represents an entirely different concept. This raises a fundamental question: are we even certain that spacetime can meaningfully be described as a field in the first place?

How do you relate neutrino detection with dark matter detection ?

Neutrinos are Standard Model particles that have been studied since the 1930s. While we know a lot about them, many of their properties remain uncertain. Interestingly, neutrino research overlaps with the search for dark matter. If dark matter consists of particles that interact very weakly with ordinary matter, similar to neutrinos, this could explain why it has eluded detection for so long. Consequently, many of the strategies used to detect neutrinos can also be applied to search for dark matter, and vice versa. In that sense, the two stories of neutrinos and dark matter are deeply intertwined and quite complex. I often tell my students that any concept or idea you can apply to neutrinos can usually be applied to dark matter and vice versa. In other words, if you can think of a model or approach for neutrinos, it often works just as well for dark matter and vice versa. This is only true at a surface level. At a deeper level, much remains uncertain. Dark matter could be strongly interacting, or it might not even be a particle.

Many people feel that progress in the field of dark matter has slowed in recent years. What is your perspective on this?

It depends on perspective. If you think of searching for something, like losing your keys, and you spend 20 years trying different methods, some might say the search has made progress, while others focus on the fact that the key hasn't been found. Similarly, over the last 40 years, the approaches to detecting dark matter have advanced tremendously, even though we still don't have a definitive answer. This shows that nature has been uncooperative. Researchers must understand that this is a struggle between human effort and nature's secrets, sometimes, if nature doesn't reveal something easily, there is only so much we can do.

Is it also true that the pace of progress in theoretical research today seems slower than it was in the past?

Yes, it does seem slower than before. But why is that? The leaps we can make in theoretical physics are often easier than those in experimental physics. Right now, we are at the very edge of what current technology allows. Experiments are pushing the limits of what humans can achieve, so while we have many ideas, testing them is extremely challenging. This is not about lack of effort. This is the absolute frontier of what is possible today. Thousands of brilliant minds are working tirelessly to improve technology, increase sensitivity, and push the boundaries in order to discover the next big breakthrough in fundamental physics.

Previously, theoretical and experimental progress often went hand in hand. Ideas flowed freely, and experimentalists were able to build cyclotrons, accelerators, underground detectors, and even instruments on balloons to test them. When something unexpected was discovered, theorists would propose explanations, and experiments would follow to test them. Today, however, experimental progress has slowed. We have reached a kind of logjam because building new experiments at the required scale and sensitivity has become extremely difficult. Come on, building a collier is not a joke, right? To make progress, you might need to build something as massive as a tunnel with 27-kilometer circumference and secure enormous funding. So it is true that experimental progress in my generation has slowed dramatically. But what can we do? It is simply not in our hands.

"We are at the very edge of what current technology allows..... Thousands of brilliant minds are working tirelessly to improve technology, increase sensitivity, and push the boundaries in order to discover the next big breakthrough in fundamental physics."

We are doing our best to extract as much insight as possible from the experimental data available from smaller-scale experiments.

What do you think the future holds for this field? How do you see upcoming developments and discoveries shaping the next phase of research?

One noticeable trend is that many researchers are now moving into astroparticle physics. I myself started as a collider physicist and earned my PhD in that field, but my shift was driven by interest rather than social reasons. At the same time, a broader wave has been occurring: many people working in collider physics are transitioning to astroparticle physics. This is largely because the era of building progressively larger colliders looks uncertain. Twenty years ago, we knew the next big collider would be the Large Hadron Collider, and the community was aligned toward that goal. Today, there is discussion in places like Europe, Japan, and China about new colliders, but nothing is definite, making the future less clear. It almost seems that astroparticle physics is becoming the main focus. The LHC is still taking data, and smaller colliders and precision machines continue to measure important quantities, so experimental work is ongoing. However, in the long run, data from space appears to be more promising because it offers a much larger playground. By astroparticle physics, we mean studies that include cosmology and the measurement of various properties of the universe using satellites and telescopes. This also involves underground detectors and numerous small to mid scale experiments that together probe fundamental physics. This appears to be the direction the field is heading.

What we really need now are good experimental ideas. The field is not limited by a lack of theoretical models. There are already plenty available, waiting to be tested. Occasionally, new models appear, but that has mostly been the work of the previous generation. Our main challenge is to develop new strategies for testing these models, designing experiments, and finding innovative ways to extract meaningful data.

What advice would you give to undergraduate students who are interested in pursuing research on dark matter?

My advice for students applies not just to dark matter research but to any field. In our country, there is a tendency to focus heavily on reading and understanding theory before doing anything. I would suggest a different approach. Start doing. Begin with calculations, work on problems from textbooks, or try to understand and reproduce results from research papers. Even second or third year students can tackle certain papers. It may take weeks, but in the process, you will learn a tremendous amount. This hands-on practice is how you truly get into research, not by reading alone. Of course, taking up a summer project can also be valuable, but the key is to actively engage with calculations and real problems.

"Begin with calculations, work on problems, or try to reproduce results from research papers. This is how you learn deeply."

So, what exactly is our current understanding about dark matter ?

We don't know much. The possibilities are very vast and that is why we should keep an open mind and look for it with many different things. I have some theoretical biases, like, you know, I personally would like that matter to be a proper particle with certain properties that also solve other problems and things like that. Everybody has their own favorite dark matter model, but if you work on only that you are, you're not gonna go anywhere. So my various students work on various possibilities. One student works on dark matter that is so light that it is like a wave across galactic scales and uses pulsar timing arrays.

Another of my students looks for dark matter that is more like a particle and look for them in underground detection and another student does the black hole stuff. So we, we cover the whole range basically. We just come with different strategies because we do not know what is the model that works.

So you mean we do not have a model which explains most of the data ?

No. The real problem is that we have thousands of models that can explain the available data. There are roughly five key pieces of evidence for dark matter, and many models are capable of satisfying them. So the main question becomes, what general properties must dark matter have to fit these observations? These broad requirements can be met by a vast number of models, which makes it very difficult to pinpoint the correct one. The challenge is not a lack of ideas but rather having too many of them and not knowing which one truly works. We can only figure it out by doing experiments where we can select a set of models over others saying that nature has cast it's vote.
That's how science goes.

This might not be explicit to your research. How do you think the current outburst of AI will impact your research?

In Theoretical physics, AI is having a lot of impact. Even in the theoretical-experimental interface and all over the place it is being used a lot. Even in astrophysics and astroparticle physics, it has a lot of uses for example, mapping the stars. A lot of people use AI to pick out different patterns in the maps and tell phenomenon like lensing etc. People are also putting a lot of effort into why is the AI finding something as an optimal thing. AI has become like a black box where you put in some information and get analyses and interpretations. We in general do not like a black box like that. So people are making a lot of effort to even understand, make it not so unsupervised to understand what is behind the black box, what is the machine actually learning?

Apart from all these things, since the discovery of the Higgs Boson, there is some hype about God particle. Can you explain what it actually is?

The standard model is basically a framework where you explain the various fundamental forces. It explains the Strong force, the Weak force and the Electromagnetic force. The standard model unifies the Electromagnetic force and the Weak force into Electro-Weak force. In that framework, you embed various particles that feel these force. Forces carry a very special meaning here in field theory where they are associated with symmetries. So now you embed all these particles and the give them all the correct role. Now, what you see is that because of this very intricate structure, which is like a jigsaw puzzle, there are various symmetries that we have to put together and then the particles have to feel only some combination of symmetries for them to get their identity. Because of this intricate structure, once you put all the particles in the standard model, you find that no particle has a mass. Everything is massless including the particles which are supposed to have mass. It's hard for me to explain the reasons unless we build up the field theory. You can refer to my NPTEL lectures for more details. So take my word for it. It has to do with symmetries like gauge symmetry because of which the particles are forbidden to have mass.

There are a couple of fixes to this. One way to fix this is to introduce a field that interacts with this particles. That is how we get the Higgs Field. This Higgs field is an agent that breaks some of the symmetries like the electroweak symmetry in order to give us the electromagnetic force and the weak force. Due to this symmetry breaking, we see that some particles pick up mass. One more interesting consequence of this, is when the breaking happens, these things get mass, but we also get one more particle that was not in the standard mode previously. That particle is the, this is the Higgs Boson which is very hard to find. The Higgs field contains four fields and three of them become the maasses and one field becomes the Higgs Boson.

Since the Higgs Boson has some mass, do you mean the Higgs field also interacts with the Higgs Boson ?

No. Higgs Boson is a part of the Higgs Field. It does not interact with the Higgs Field.

Let us kick that question back and ask, how does the higgs field acquire mass? If anybody answers it, let know. This is a question. There are two ways to ask that question. One is, why shouldn't it get a mass? Because in a field theory, when you write a scalar field, it will pick up a mass. But why only this mass and not anything else? It comes down to asking what is the mechanism by which the Higgs field comes that nobody knows an answer to. This is an incredibly deep problem. Something in nature has determined the mass and other scales that allow all of these to exist, yet we have no idea what sets them. This is more like an effective field theory where we put different fields by hand and show experimentally it predicts all the effects but the causes of this is still a mystery.

MODELLING NEURONS: THE HODGKIN-HUXLEY MODEL

Ritabrata Saha

Introduction :

The year was 1952 when Sir Alan Hodgkin and Sir Andrew Huxley published a groundbreaking mathematical framework that would become the cornerstone of modern biophysically based neural modeling. Their work emerged from a series of elegant electro-physiological experiments conducted in the late 1940s and early 1950s on the squid giant axon—a nerve fiber remarkable for its exceptionally large diameter of approximately 0.5 mm. This unique feature, which evolved to enable rapid conduction of action potentials for escape responses, also offered a technical advantage for researchers. Unlike the much thinner axons found in most nervous systems, the squid giant axon's size allowed for experimental manipulations that had previously been impossible.

They developed a detailed set of coupled differential equations that captured the ionic basis of the action potential. This mathematical formulation, now known as the Hodgkin-Huxley (HH) model, revealed its true power when the researchers used a hand-cranked mechanical calculator to numerically integrate the equations—successfully reproducing all the key biophysical features of the action potential. Their contributions not only revolutionized our understanding of nerve function but also set a precedent for quantitative modeling in neuroscience. In recognition of this landmark achievement, Hodgkin and Huxley were awarded the Nobel Prize in Physiology or Medicine in 1963, which they shared with Sir John Eccles for his pioneering work on synaptic transmission.

A General Mathematical Framework :

In their seminal paper on the biophysical basis of the action potential, Hodgkin and Huxley (1952) modeled a segment of squid giant axon using an equivalent circuit similar to that shown in Fig. 1. It had two major components: One represents the flow of current due to capacitive charge difference, the other represents the flow of ions due through ion channels, namely sodium and potassium, and it also has a leakage current due to chloride ions. The capacitive current $I_c = C_m \frac{dV_m}{dt}$. One thing to note is that q(t) is related to the instantaneous membrane voltage $V_m(t)$ and the membrane capacitance C_m by the relationship $q = C_m V_m$.

Thus,

$$C \frac{dV_m}{dt} + I_{ion} = I_{leak}$$

$$I_{ion} = \sum_i G_k(V_k - E_k)$$

$$I_{\text{ion}} = G_{\text{Na}}(V_m - E_{\text{Na}}) + G_K(V_m - E_K) + G_L(V_m - E_L)$$

Here G_k is conductance.

In the Hodgkin-Huxley (HH) model, ion channel conductances arise from microscopic gates that switch between two states:

- **Permissive (open) :** Allows ion flow.
- **Non-permissive (closed) :** Blocks ion flow.

For an ion channel to conduct, **all its gates must be permissive**. The probability p_i represents the fraction of gates of type - i in the permissive state at time t, while $1 - p_i(t)$ are non-permissive.
Gate transitions are governed by voltage-dependent rates:

- $\alpha_i(V)$ **:** Rate from non-permissive to permissive.
- $\beta_i(V)$ **:** Rate from permissive to non-permissive.

These transitions follow **first-order kinetics**, and the change in the probability $p_i(t)$ over time is given by:

$$\frac{dp_i}{dt} = \alpha_i(V)(1 - p_i) - \beta_i(V)p_i$$

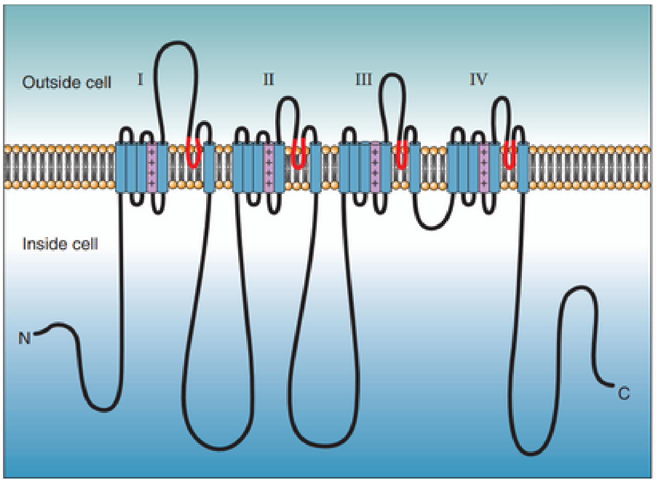
At a constant membrane voltage V_r , the system reaches a steady state as time goes on. At this point, ,$\frac{dp_i}{dt} = 0$, and the steady-state fraction of gates in the permissive state becomes:

$$p_{i,\infty} = \frac{\alpha_i(V)}{\alpha_i(V) + \beta_i(V)}$$

The system also has a **time constant $\tau_i(V)$**, which tells us how quickly it approaches this steady state. The time constant is given by:

$$\tau_i(V) = \frac{1}{\alpha_i(V) + \beta_i(V)}$$

This time constant determines how quickly the fraction of permissive gates approaches the steady-state value in response to a voltage change, following an exponential time course.



Standard HH Conductance Formulation :

The maximum conductance for a given ion channel type is defined by a constant $\bar{g}_k$, which represents the conductance when all the gates are in the permissive state.Equations above used general notation, but to match the standard Hodgkin-Huxley model more closely, the variable p_i is replaced by specific symbols for each gate type.

For example, in the HH model:

- Sodium conductance is modeled with three m-type gates and one h-type gate.
- Potassium conductance is modeled with four n-type gates.

Well, the question arises how did the HH model know exactly that there were 4 and 3 gates? The physiological model of Neuron wasn't discovered yet. That is a question we will address in the next section.

The sodium conductance becomes

$$G_{\text{Na}} = \bar{g}_{\text{Na}} p_m^3 p_h = \bar{g}_{\text{Na}} m^3 h$$

The potassium conductance becomes:

$$G_K = \bar{g}_K p_n^4 = \bar{g}_K n^4$$

The total ionic current in the HH model is:

$$I_{\text{ion}} = \bar{g}_{\text{Na}} m^3 h (V_m - E_{\text{Na}})$$

$$+\bar{g}_K n^4 (V_m - E_K) + g_L (V_m - E_L)$$

The gating variables evolve over time according to:

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m$$

$$\frac{dh}{dt} = \alpha_{h(V)}(1-h) - \beta_{h(V)}h$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

Rate Constants :

This is what H-H had to say about their predictions. However their model based purely on numerical methods did provide a suitable basis for the actual physical model.

Note : *For the sake of illustration we shall try to provide a physical basis for the equations, but must emphasize that the interpretation given is unlikely to provide a correct picture of the membrane.*

Potassium Conductance :

Let's try to quantify the potassium equations.

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

Initially, at $V_m = 0$, n reaches a steady - state value given by:

$$n_\infty(0) = \frac{\alpha_n(0)}{\alpha_n(0) + \beta_n(0)}$$

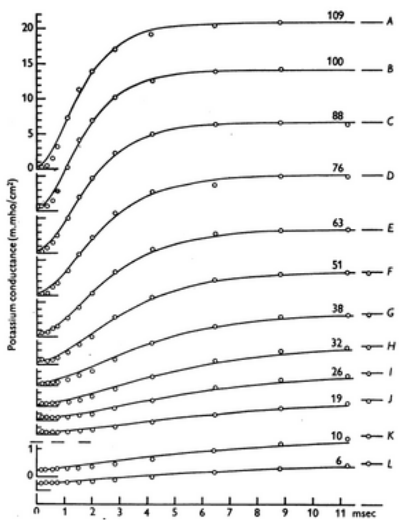
The solution to equation (14), with the appropriate boundary conditions, $n = n_o$ when t = 0, takes the exponential form:

$$n(t) = n_\infty(V_c) - [n_\infty(V_c) - n_\infty(0)]e^{-t/\tau_n(V_c)}$$

From this, we get

$$g_K = [(g_{K_\infty})^{\frac{1}{2}} - [(g_{K_\infty})^{\frac{1}{2}} - (g_{K_0})^{\frac{1}{2}}]exp(-t/\tau_n)]^4$$

where g_{K_∞} is the value which the conducatnace finally attains and g_{K_0} is the conductance at t=0. This equation describes how n evolves over time after a step change in membrane voltage. One could fit this curve to experimental data to estimate $n_\infty(V_c)$ $n_\infty(V_c)$ and $\tau_n(V_c)$



Rise of potassium conductance associated with different depolarizations. Circles represent experimental data points on Axon 17, temperature 6-7 ̊ using observations in sea water.

$(g_{K_o} = 0.24mS/cm^2)$

Thus there is reasonable accuracy between experimental and theoretical data leading to the prediction that indeed potassium pump has 4 gates.
The rate constants α_n and β_n were calculate by curve fitting.

$$\alpha_n = 0.01(V+10)\left[\exp\left(\frac{V+10}{10}\right)-1\right]$$

$$\beta_n = 0.125\exp\left(\frac{V}{80}\right)$$

Sodium Conductance :
We might suppose that sodium conductance is determined by two variables, each of which obeys a first-order differential equation. These two alternatives correspond roughly to the two general types of mechanism mentioned in connection with the nature of inactivation

The formal assumptions made are:

$$g_{\mathrm{Na}} = m^3 h \bar{g}_{\mathrm{Na}},$$

$$\frac{dm}{dt} = \alpha_m(1-m) - \beta_m m,$$

$$\frac{dh}{dt} = \alpha_{h(1-h)} - \beta_h h,$$

where $\bar{g}_{\mathrm{Na}}$ is a constant and the α's and β's are functions of V but not of t.

These equations may be given a physical interpretation if sodium conductance is assumed to be proportional to the number of sites on the inside of the membrane that are occupied simultaneously by three activating molecules but are not blocked by an inactivating molecule. The variable m represents the proportion of activating molecules on the inside, and 1 - m represents the proportion on the outside. The variable h is the proportion of inactivating molecules on the outside, and 1 - h is the proportion on the inside. The parameters α_n or β_n and α_n or β_n represent the transfer rate constants in the two directions.

The solutions of previous equations which satisfy the boundary conditions $m = m_0$ and $h = h_0$ at t = 0 are

$$m = m_\infty - (m_\infty - m_0)\exp(-t/\tau_m),$$

$$h = h_\infty - (h_\infty - h_0)\exp(-t/\tau_h),$$

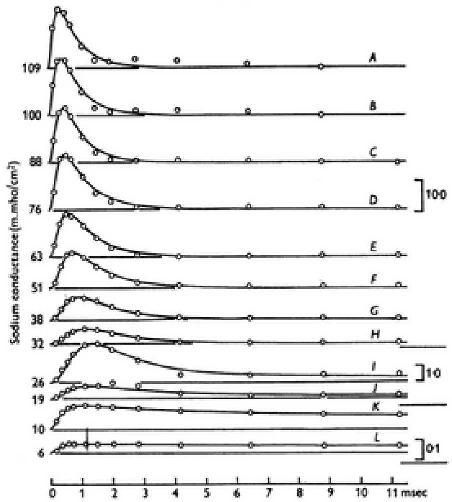
where

$$m_\infty = \frac{\alpha_m}{\alpha_m+\beta_m} \quad \text{and} \quad \tau_m = \frac{1}{\alpha_m+\beta_m},$$

$$h_\infty = \frac{\alpha_h}{\alpha_h+\beta_h} \quad \text{and} \quad \tau_h = \frac{1}{\alpha_h+\beta_h}.$$

In the resting state the sodium conductance is very small compared with the value attained during a large depolarization. We therefore neglect m_o. The expression for the sodium conductance then becomes

$$g_{\mathrm{Na}} = \bar{g}_{\mathrm{Na}}\left[1-\exp\left(-\frac{t}{\tau_m}\right)\right]^3 \exp\left(-\frac{t}{\tau_h}\right)$$



Change of sodium conductance with different depolarization. Circles represent experimental estimates of sodium conductance and the solid line represent theoretical observations.

Thus again it is indeed very accurate match. The other values were also found by curve fitting as usual.

$$\alpha_{\hbar} = \frac{h_{\alpha\beta}}{\tau_{\hbar}},$$

$$\beta_{\hbar} = \frac{1-h_{\alpha\beta}}{\tau_{\hbar}}$$

$$\alpha_m = 0.1\ (V+25)\Big/\left(\exp\frac{V+25}{10}-1\right),$$

$$\beta_m = 4\exp\left(\frac{V}{18}\right)$$

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HIGH ENERGY PHYSICS, NON-COMMUTATIVE SPACES AND SU(2)

Sriraj Chandra, Gunda Sai Vinay

Professor Sachindeo Vaidya is a faculty member at the Centre for High Energy Physics (CHEP), Indian Institute of Science, Bangalore. He obtained his Ph.D. from Syracuse University in 1998 and joined IISc in 2012. His research focuses on theoretical high-energy physics, particularly on quantum field theory, non-commutative geometry, and deformations of space-time symmetries. He investigates the role of quantum groups, twist deformations, and geometric approaches to field theory in understanding the structure of space-time at the Planck scale.

Let's begin this interview. So, Professor, what has been your experience with undergrads in the courses you have taken?

The UG programme is now almost 15 years old. Although I have not taught an actual undergraduate course, I have had undergrads enroll in the courses I have taken throughout this 15 year period. By and large, the undergraduate students in my class are hardworking and clever. I'm happy that such an audience participates in my courses and is able to engage in them.

So, as you yourself said, it has been only 15 years since the inception of the undergraduate program. How do you think the undergraduate population here has affected the institute, given that they were not here until quite recently?

The undergraduates come to the institute with a different level of preparation i.e. just after finishing higher secondary. They have a different level of subject knowledge as compared to the other students here, which allows many of them to have fresh perspective on old questions. Although it might not suffice to solve the questions, it sometimes allows one to make a lot of progress. Thus, having such young minds in the institute proves pretty useful. They bring new viewpoints to discussions on age-old ideas, which benefits research all around.

Dr. Sachindeo Vaidya

Throughout your whole tenure at IISc, you have taught and conducted research simultaneously. How do you view these activities as opposed to each other?

Well, I enjoy both of them in different ways. There is no direct influence of one on the other. Teaching energizes me i.e. it changes my intellectual energy level. That often translates into an impact on my research work. There is almost never a direct connection between what I am teaching and my research, but the indirect influence of teaching positively affects my research.

Do you prefer simultaneous teaching and research to just research?

I would be unhappy if my teaching was reduced to zero, because, like I said, you meet young people and they ask things which sometimes brings a very fresh perspective to the question. Also, I feel it is a part of my professional and moral responsibility to impart to young people the knowledge I have gained, which again has been due to some excellent teachers.

For aspiring physicists like us, how would you say HEP is as a field in terms of research potential?

As you may know, the field of HEP has an old history. Some may trace it back to the discovery of the proton/electron i.e. about a century ago. But HEP, as a profession, as employment for a large no. of researchers, owes its real development to the need for countries for developing atomic weapons. So, the funding for HEP increased enormously from the 40s and 50s, all the way through the 60s and 70s. India has had an atomic energy programme since this period. As we needed to understand subnuclear physics much better for such development, governments felt that they needed skilled intellectual manpower because the problems were very hard. This led to a high amount of funding for HEP, both in terms of training manpower as well as for doing experiments. Many of the colliders that came were built in the 50s and 60s, and in time, evolved into their current forms. This is, if you like, the reason why HEP became a profession rather than just a subject. Today, because of a variety of funding reasons, the global interest in HEP is decreasing. That, of course does not mean that the questions that are of interest in HEP have been answered. On the contrary, we are still far from having satisfactory answers to such questions. HEP plays an important role in understanding astrophysics, and cosmology, especially early-universe. So, there are fundamental and deep questions that are appealing not just to us as scientists, but even ordinary people who wonder about stars, the universe and how it began etc. Earlier, these questions were considered philosophical, but today, due to the advancement of science, these are questions that can be answered in a more systematic and scientific way. So, there are questions in HEP that will have a significant impact on other domains of physics. I cannot say whether this is the right time for young people to do HEP, but, I will say that you should go with what you really like. In that case, even if it doesn't work out, you will not, at a later stage, regret that you had not done HEP when you had wanted to do so as you still love it. If it works out, the subject will be invariably enriched if a person highly interested in it joins its development. This is the only advice I would like to impart in this general context.

“It is not enough to have a good teacher. There must be an internal thirst in you. Something that you have seen or wanted to understand must really bother you, to make you question its reason.”

“Mathematical rigour sometimes provides you guiderails on that metaphorical path so that you don't fall off the cliff. ”

What would you say about the mathematical rigour required for working in theoretical HEP? Also, what kind of mathematical ability do you think is needed in theoretical HEP as compared to other branches of physics?

I think of maths as useful and the role of rigour is only as a guiderail. All research, essentially, is about walking in uncharted territory. If you would like a visualization, it is equivalent to walking on a mountain path. Mathematical rigour sometimes provides you guiderails on that metaphorical path so that you don't fall off the cliff. It is to this extent that mathematical rigour is useful. It is often seen that people make conjectures and guesses, which then fall apart because they had forgotten some simple piece of mathematics that they had overlooked in the process of building that conjecture. Thus, rigour is mostly useful in terms of maintatining mental discipline. To understand nature, mental discipline is required, but as physicists, just achieving mental discipline is not the end. Using that successfully to understand nature better is our primary goal. Now, in regard to the following question, most people that I have come across at IISc are very intelligent. Almost without exception, I have found them to possess the necessary abilities. If there is a question in physics that engages you with burning curiosity, it will invariably lead you to learn and understand the mathematics behind it. This is more than enough for physicists. In fact, this is how most physicists build the mathematical background for their work, as opposed to having a fully equipped mathematical arsenal beforehand.

Now, it is time to ask perhaps the most cliche question in physics interviews. How would you describe your love for physics as a subject? Especially, what had guided you to this path of a physicist?

I did my schooling in a village school in Goa, where I had the fortune of having a very good math and science teacher, in spite of it being a very small school. The population of the village was a few thousand at that time. I benefitted immensely from talking to this man. He encouraged questions and discussions in his class. This, or indirectly he, was the reason I got interested in physics and mathematics during middle and high school. Then I appeared for the IIT entrance examination, and went to IIT Kanpur to pursue a 5 year integrated M.Sc. in Physics. There, I again happened to meet some excellent teachers. At this point I must add, that to really appreciate physics, you must be mentally prepared. It is not enough to have a good teacher. There must be an internal thirst in you. Something that you have seen or wanted to understand must really bother you, to make you question its reason. When one is prepared like that, then only a good teacher can take you far into the subject. Otherwise, if your curiosity has not been aroused, irrespective of the teaching, you are bound to fall short of your mark. I speak from personal experience when I say this. I have seen that when I have not been sufficiently curious, taking a class with a good teacher has had a very minimal effect on my knowledge of the subject. On the other hand, when I have been extremely curious, taking such a class has advanced my knowledge and understanding tremendously. Thus, the preparation of the student matters. By preparation, I don't mean mere technical strength, I mean the presence of that burning curiosity. If you don't have that, at the most, you might do well in that class and leave satisfied with a good enough grade. That would not have changed you as a physicist, as a student of science. This has been my experience as a person who was in both of these situations for various classes. This phenomenon also repeated during my PhD, where I had subjects I was not that interested in as well as those I was extremely curious about. I finally ended up doing my PhD in the latter.

Now, I would like to ask you a more technical question. As most of your research is in mathematical physics, how often do you encounter SU(N) in your line of work?

As you must know, SU(N) is the group of NXN unitary matrices with determinant 1. I would say that my intersection with SU(N), in regard to my personal research, is periodic. There are times when I have to deal a lot with SU(2) and SU(3), and occasionally SU(4). In my research, I mostly deal with SU(N) at the lower end of N. SU(N) for general N is a fascinating group, rich with questions that shape large areas of physics and mathematics. For instance, QCD is described as an SU(3) gauge theory.

We'll move on the next question. For people who might want to transition from HEP to other fields of physics or vice-versa, how hard do you think it might be? How could such a change be handled?

A lot depends on the stage at which this transition is happening. If this is a young undergraduate, it is not that he/she is deep inside HEP. The transition is not difficult at all in such cases. Even in the case of a senior researcher, the difficulty of such a transition depends on the curiosity he/she has for the new field. If that thirst is there, such a transition becomes a lot smoother than if it were not there. The curiosity itself guides them to learn things at an accelerated pace and be at peace in the new field. Thus, the best way to handle such a change cannot be forced - if there is curiosity, he/she will find the way.

What have been your research interests and how have they evolved during your time as a researcher? Also, how would you describe your academic evolution, from the undergraduate level to your current position?

As I mentioned, I did my integrated M.Sc. from IIT Kanpur. At that time, it was very popular for people to want to do HEP just because of the environment we were in. I had no idea about HEP then. I knew that there was a collider in Europe and one in standford. I had no idea what they did. I just had a rough idea about the Standard Model. Around a decade before that, Salam, Glashow and Weinberg won the Nobel Prize for their famous model. I had heard of all these things but I was not engaged in what they meant. Despite that, I had good grades and was able to get a scholarship to go to the U.S.. My curiosity levels were enormously enhanced when I met my PhD supervisor at Syracuse University. My PhD thesis is a work on topology and geometry but in molecular physics. It turns out that such mathematics is important in understanding the nature and spectra of molecules. As a result of doing such work in my doctoral thesis, I have been since then biased towards understanding the role of topology and geometry in HEP. In general, I am curious as to how these ideas of topology and geometry intersect and impact all parts of physics such as CMP and classical physics. The role of these two mathematical tools in physics and how they help us understand and simplify different problems in physics has been my guide in how I choose my work and my research problems.

As I have said earlier, I did my schooling in a village in Goa. I stayed in a hostel in Panji to finish my higher secondary education. Then, I went to IIT Kanpur to pursue a 5 year integrated M.Sc. in Physics, and to Syracuse University in the US for my PhD. Following that, I did two postdoctorates - one at TIFR, the other at UC Davis. Finally, I have been in IISc since 2003. Throughout all of this, it is important to remember that fundamentally, one is always a student. What changes is the nature of one's responsibilities, especially scientific responsibilities. As one grows senior, there is an increase of scientific responsibilities. The reason that happens is that seniority brings enough experience for one to be able to guide young individuals. Other than that, there are a lot of administrative responsibilities. My doctoral thesis was on topological aspects of the Born-Oppenheimer approximation.

As you may know, the Born-Oppenheimer approximation allows you to study molecules. The geometry and topology involved in it began to be appreciated after the work of Michael Berry who showed that if such aspects are ignored, simple phenomena in molecular physics cannot be explained. Although one learns of this approximation in context of molecular physics, it extends from molecular physics to nuclear physics, to even String theory, where if certain things cannot be understood using the approximation, they cannot be understood at all. I saw how important these two mathematical tools were in improving the conceptual as well as phenomenological aspects of several physical systems. After my PhD, I took an interest in looking at monopoles in the context of non-commutative physics. I worked on gauge theories on non-commutative spaces as well as on instantons. As a natural part of that discussion, we also looked at what happens to the notion of identity of two particles. We know, for example, that any two electrons are identical. Thus, their combined wavefunction is antisymmetric under exchange. What is remarkable is that, if the wavefunction is antisymmetric in one Lorentz frame, it has to antisymmetric in all Lorentz frames. If you could find a Lorentz frame in which it is not antisymmetric, that wavefunction cannot correspond to identical fermions. So, the notion of identity of particles, incorporated by the antisymmetry of the wavefunction, has to be Lorentz-invariant. Some of the work I did had to do with the idea of extending this idea of identical particles to non-commutative field theories. It's very interesting, and is driven by geometric and topological questions. Around that time, I started to get interested in questions of entropy as a result of my involvement with representation theory while trying to work with identical particles. As a result of my interest in entropy, I got re-interested in QCD. Over the past few years, my primary work has been in understanding aspects of QCD at extremely low energies. QCD is very difficult to understand at low energies because the coupling becomes very large, and our standard tools of perturbation theory fail completely. On the other hand, perturbation theory works very nicely for QCD at high energies. However, most of interesting QCD phenomena happen at low energies, and that is where I currently am engaged in as a researcher. I believe this can be an apt description of my evolution from someone with his doctorate in topological aspects of Born-Oppenheimer approximation to my current research interest in low-energy QCD.

Professor, can you give us a brief introduction to non-commutative geometry? It was one of the primary topics listed under your research interests.

Our usual understanding of geometry is that we have a space described by smooth functions. For example, you can look at scalar and vector fields on the space. You can also discuss, say metrics on the space etc. By and large, our physical world informs us that if you make an approximation that this space is smooth, i.e. described by sufficiently smooth structures, this approximation takes you quite far. It works excellently for a large part of physics, from say, classical mechanics up till general theory of relativity, the geometry on the tensorial structures being smooth. From a physical point of view, at some level, this is an approximation in the same sense as the fact that modelling the air around us or the water in a river as a continuous fluid is an approximation. The latter is a perfectly reasonable thing to do for describing large classes of phenomena in fluid flow. But, as soon as you want to discuss phenomena which are comparable to the mean free path, you find that your description fails in the following sense: Whatever predictions you make on the basis of a theory that requires to describe it as a smooth fluid, those predictions do not match the corresponding experimental results. There is a big mismatch. Then you realize that something else is going on at that level and you cannot use this approximation. Now, one moves into the molecular or atomic theory of fluids. That is the right way to think of fluids when the length scale of the phenomena is shorter than the mean free path. There is a similar situation in the part of physics which deals with gravity. It is similar in only one aspect i.e. in gravitational theories, when you want to include quantum mechanics, there is a natural length scale you can build out of Planck's constant, Newton's constant and the speed of light, known as Planck length. So, this is a warning flag for all researchers who want to understand quantum effects of gravity. This is because for events which occur at a length scale comparable to Planck length, one cannot use gravity as a smooth structure. It is one thing to say that such a description cannot be used, and a completely different thing to successfully replace it. In the case of the example with air or water, we had experiments to guide us, for example, in the case of Brownian motion. There are many experiments to tell us how to approach this problem when they cannot be explained by the Navier-Stokes equation. Unfortunately, in the case of gravity, we have no experiments of that nature. We do not have any experimental guide to help us on the conceptual theoretical framework for describing quantum phenomena in gravitational domains. One of the proposals for such a conceptual framework goes by the name of non-commutative geometry. Here, the idea is that, if you are on a smooth manifold, functions commute i.e. the order of multiplication of functions does not matter. In non-commutative geometry, this is no longer true, which sort of captures, in a precise sense, which requires some sophistication to describe, the possibility that coordinates may not be commutative and thus the space need not be smooth in the traditional differential geometric sense. This is, if you like, the origin of interest in non-commutative geometry for physicists. Non-commutative geometry is, of course, a very well developed branch of mathematics. This is the reason why physicists are interested in non-commutative geometry.

For novices like us, could you provide a familiar description of NCG?

Such a description can be, say, in terms of the phase space of a one-dimensional particle, which is thus two-dimensional, with x and p as the two dimensions. In classical mechanics, dynamics is described on the phase space by the Hamiltonian, which is a function on the phase space. If you keep changing the Hamiltonian, you keep on describing different systems. The phase space is the playground, and the Hamiltonian is the referee, if you like, who sets the rule of what happens. In classical mechanics, one is interested about functions on the phase space, such as angular momentum or energy i.e. about their dynamics for a given Hamiltonian. Now, one thing is that these functions commute. For example, it does not matter in which order you multiply L_x and L_y in classical mechanics. If you like, classical dynamics is characterised by the commutative algebra of functions on the phase space. You write functions on the phase space, and the rule by which you multiply these functions is commutative, thus leading to a commutative algebra. What happens in quantum mechanics is that you realize, in phase space, functions no longer commute. L_x multiplied with L_y is not the same as L_y multiplied with L_y. In this sense, quantum mechanics is essentially described by a non-commutative algebra. To a large extent, you may even take this as a definition of quantum mechanics. If the dynamics is described by a commutative algebra, it must be classical. If it is described by a non-commutative algebra, it has to be a quantum system. This is a way to think about the role of non-commutative algebras. One can extend this idea to the configuration space. In the phase space, in quantum mechanics, x_1 and x_2 do not commute, but x_1 and x_2 commute. You can imagine extending this discussion to a situation where even x_1 and x_2 do not commute. Of course, at this level of description, it is just a game. One

“We do not have any experimental guide to help us on the conceptual theoretical framework for describing quantum phenomena in gravitational domains. One of the proposals for such a conceptual framework goes by the name of non-commutative geometry.”

might ask the use of such a description. There is actually a very important situation where this is useful i.e. in the description of the quantum hall effect. In the classical hall effect, you have a z-directed magnetic field and electrons in the x-y plane. If one looks at the whole plane, the phenomenon is nothing more than the circular motion of electrons due to the magnetic field. In the quantum hall effect, you can show that the x-coordinate of the centre of that circular path and the corresponding y-coordinate do not commute, just by using ordinary quantum mechanics. These are merely spacial coordinates, not even operators like x or p. In this sense, a non-commutative plane arises very naturally when you discuss certain aspects of the quantum hall effect. This is one simple example where NCG comes out in a natural manner. Now, whether it is natural or not in the case of quantum gravity is really an open question as we have no experimental guidance. It is the hope for a large number of physicists that NCG might work, and is perhaps one idea to try out in order to describe quantum physics at Planck scale. I hope this answers your question.

How would a photon look like in a non-commutative space, and how would it be as compared to that in a commutative space like ours?

To answer your question, we would have to go back a little. Photons are excitations in the theory of quantum electrodynamics. The first thing one has to do is to figure out how to write down QED in NC spaces, and then look for excitations in such QED. To the best of our knowledge i.e. available experimental signatures, the spacetime that we have access to energetically is commutative. Even for the highest energy scales achieved till date, there has been no evidence for non-commutativity. So, when you want to ask about imagining photons in NC spaces, I would say the right way to formulate this question is, ``Can we write down a theory of QED on a non-commutative space such that when the photon wavelength is very large, it reduces to ordinary QED?''. Ordinary QED explains everything around us. If such a theory does not look like ordinary QED at the large wavelength scale, you immediately know that the theory has to be wrong. This is the way to try to think about photons in NC spaces. Now, if there is a theory which satisfies these requirements, we can ask how a photon looks like when the wavelength is of the non-commutative scale. Interesting things can happen at that scale but again, those would only be accessible if you can conduct experiments at that scale. I think this is a way you can think about photons in NC spaces.

How do you visualize a geometry in which the x and y-coordinates do not commute?

One can start from even the question of visualization of a geometry where x and y commute. Visualization comes from the idea of the phase space functions, which correspond to experimentally measurable observables. A series of experiments on the same would lead you to conclude indirectly that these functions are commutative. This would indicate that you are in the classical domain. Similarly, there should be a class of experiments and observables which, if correctly interpreted, would lead you to conclude that the x and y-coordinates do not commute. For example, this would lead to wave packets that cannot be shrunk to a delta function in the x-y space, due to the uncertainty in the x-y coordinates whereas in a commutative situation, this is possible.

ANDERSON - LIKE LOCALIZATION IN QUANTUM DOTS

Panchariya Chinmay Anil

Introduction to Fock Space :

Hilbert Space is a closed vector space under a norm. The quantum mechanical description, developed by Dirac and contemporary scientists and perfected by John Von Neumann, is based on Hilbert Space. But, there is an issue with it, Hilbert space is designed to handle a fixed number of particles, but in Quantum Field Theory, in intermediate steps, new particles are formed and annihilated, hence we have to modify our notion of Hilbert space to Fock Space, defined by:

$$F_\nu(H) = \mathbb{C} \oplus H \oplus (S_\nu(H \otimes H)) \oplus (S_\nu(H \otimes H \otimes H)) \oplus \cdots$$

H is the Hilbert space of a single particle, so the first term is just a complex field corresponding to no particles. The second term shows the Hilbert space of one particle, and the third term is that of two particles(constructed by the tensor product of two particles up to the symmetrization operator S_ν (dependent on the nature of the particle, etc).

To have an intuition of it, imagine a building with (ideally) infinite levels... and there are ladders to go from one level to another, these ladders are raising and lowering operators. Hence, if you start with a vacuum, if u apply ladder of momenta p_1 p_2, twice on it, you will be on level 2(third term) with two particles of momenta p_1 and p_2.

System :

In the system we are considering, there is an electron quasi-particle in a quantum dot that decays into two quasi-particles and one hole. So, the interaction Hamiltonian will have two annihilation operators (one for the decayed electron quasi-particle, one for the newly formed hole) and two creation operators(for the two formed electron quasi-particles). Our system is at a low finite temperature, so we start with the ground state and treat it as a reference point. Instead of a true vacuum, we start with the N-fermi vacuum, where there are N fermions already present in the system. Hamiltonian in Fock space is:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 = \sum_\alpha E_\alpha c^\dagger_\alpha c_\alpha + \sum_{\alpha\beta\gamma\delta} V^{\alpha\beta}_{\gamma\delta} c^\dagger_\gamma c^\dagger_\delta c_\beta c_\alpha$$

As transitions(decays) will keep on happening, some of the electrons(we will drop the quasi-particle for simplicity) will be absent from the original place of fermions in N-fermi vacuum, and will be present at some other energy levels. All such configurations will form the basis of our Fock space Ψ_N, also called Slater determinants. If m electrons are absent, then:

$$\Psi_N = c^\dagger_{\alpha_{2m}} \cdots c^\dagger_{\alpha_{m+1}} c_{\alpha_m} \cdots c_{\alpha_1} |N\rangle$$

$|N\rangle$ is N-fermi vacuum. We treat this $|N\rangle$ as reference, and do all calculations with respect to it. Hence, energy of given Ψ_N would be: $E_{\alpha_{2m}} + \cdots + E_{\alpha_{m+1}} + |E_{\alpha_m}| + \cdots + |E_{\alpha_1}|$.

We introduce notion of binary bit-string labeling, where occupied states are shown by 1, unoccupied are shown by 0. So, our N-fermi vacuum is $|11..100..\rangle$. If m=1, that is one of the 1 is replaced by 0, and it goes to other place(so 0 to 1). Then, we call it hamming distance(or simply, distance) of 2(indicating 2 placed in our bit-string has change). We define distance between two states as Since our Hamiltonian only allows 2 electron transfer between levels per decay(transition): $\langle \Psi_N | \mathcal{H}_1 | \Psi_{N'} \rangle$ equals 0, 2 or 4. (We have added diagonal part of interaction Hamiltonian to $\mathcal{H}_0$ using Hartree-Fock method).

Other important parameters :

Some other important parameters about systems are: K, V, W, Z.

- Let, we start with (N-1)-Fermi vacuum. We say, first generation is $c^\dagger_\alpha |N-1\rangle$. Third and Fifth generation as $c^\dagger_\alpha c^\dagger_\beta c_\gamma |N-1\rangle$ and $c^\dagger_\alpha c^\dagger_\beta c^\dagger_\gamma c_\sigma c_\rho |N-1\rangle$, and so on. $(2n+1)^{st}$ generation means we are having m=n(refer equation 3). K_n is number of branches(with distance 2) from generation (2n-1) to (2n+1). when n is close to N (but $n < N$), we see $K_n \approx g^3/6n$, where g is experiment-dependent quantity.

- V is root mean squared of $V^{\alpha\beta}_{\gamma\delta}$, which is defined as:

$$V^{\alpha\beta}_{\gamma\delta} = \iint dx dx' V(x - x') \psi^*_\delta(x) \psi^*_\gamma(x') \psi_\beta(x) \psi_\alpha(x')$$

(Here, we are in x-basis, this equation shows an electron from state α goes to state γ, and so on. $V(x-x')$ tells about the range of transitions. We can consider transitions to be local. Hence, $V(a) \propto \delta(a)$. V is the root mean square, considering this approximation, written in terms of diffusion operator eigenvalues. Thouless energy is an energy scale of this diffusion(from one state to other, in Fock space), such that: If the energy difference between states concerned exceeds E_c, V is considerably low(can be thought of as a resonance).

- Possible on-site energy levels are from a uniform distribution in interval [-W, W].
- Z ~ W/V. Dimensionless scale of the system.
- Structure of our problem resembles the Bethe Lattice, where only hopping to next generation is allowed.

Calculating probabilities :

Defining amplitude of a state in $(2n+1)^{st}$ generation from generation 1(of energy E), as:

$$A_n = \prod_{i=1}^{n} \frac{V}{E - E_i}$$

Defining y_i as:

$$y_i = ln\left(\frac{W}{|E - E_i|}\right)$$

$$ln(Z^n|A_n|) = \sum_{i=1}^{n} y_i$$

If $\Delta_i E \equiv E - E_i$ is from a uniform distribution over $[-W, W]$, $P(y_i) = exp(-y_i)$ for $0 \le y_i < \infty$ and sufficient algebra gives, distribution of A_n as:

$$P(|A_n|) = \frac{[\ln(|A_n|Z^n)]^{n-1}}{Z^n(n-1)!|A_n|^2}, \quad Z^{-n} < |A_n|$$

Since resonance is necessary for significant transition amplitude, we are interested in

$$p(n, C) = \int_C^1 dAP(A)$$

which is approximately (for large n):

$$p(n, C) \approx \frac{1}{(n-1)!} \cdot \frac{1}{C\ln(CZ^n)} \left[Z^{-1}\ln(CZ^n)\right]^n$$

But if none of the K_n trajectories connect to first generation with significant amplitude, we can say its amplitude is given by:

$$[1 - p(n, C)]^{K_n} \equiv exp(-f_n)$$

If such P(n,C) is much less than one(A Very less number of trajectories from 2n-1 to 2n+1 have resonant amplitude), we can approximate the result in above equation to give a simpler expression of f_n as:

$$f_n \approx \frac{KE}{\sqrt{2\pi n}\, CZ} \left[\frac{KE}{Z}(\ln Z + \ln(C/n))\right]^{n-1}$$

Hence, if there is more connectivity of state to generation 1, then f_n would be higher(even at larger n). When inside the bracket term will reach 1, the system will localize. Thus, we can say that localization occurs at : [1]

$$Z = K\, ln(K)$$

Inference :

In our system, we consider a large connectivity K, and we analyze different regimes based on the relative scaling of Z (the coordination number or effective number of resonant neighbors):

- $K \ll Z \ll K \ln K$: The system lies in a moderately connected regime. In this case, resonances typically occur only at distant generations in the tree-like structure. Let n_0 be the generation at which the first significant resonance occurs. Then, as Z increases in this regime, we have $n_0 \to \infty$ indicating that resonances become increasingly rare at shallow levels, leading to delocalization at deeper levels. This means the initial state eventually spreads over many configurations, indicating delocalization.

- $Z \ll K$: In this regime, the system has a low effective connectivity, and the branching ratio is roughly reduced to $K_{\text{eff}} \sim K/Z$. The initial state spreads to other nearby states, but the state-space exploration is sparse. Hence, the system is in a non-ergodic extended phase, where the state is extended over many configurations, but still does not cover the full Hilbert space uniformly. For Z=1, ergodicity is restored.

Talking about the same phenomenon in terms of energy :

Delocalization occurs at

$$E^{**} = (\lambda b_d)^{-1/2}\sqrt{\Delta E_c / ln\ g}$$

And the transition from 1. to 2. occurs at:

$$E^{*} = (\lambda b_d)^{-1/2}\sqrt{\Delta E_c}$$

where g is as explained in section 3, and b_d is defined as:

$$b_d^2 = \frac{2}{\pi^2} \sum_{m \neq 0} \frac{\gamma_1^2}{\gamma_m^2}$$

E is the energy of a quasi-particle.

- $E < E^{**}$
  At such a low energy, our system is localized. Hence, instead of the initial state spreading into mixed superposition states of many, our many-body states are close to pure states (Slater determinant). If observed experimentally, we see sharp peaks corresponding to these states.

- $E^{**} < E < E^{*}$
  This is an intermediate energy case, where delocalization has just started, but resonances are between distant generations. Hence, the probability that we found a cluster of states experimentally is very small.

- $E^{*} < E$
  Delocalization has occurred. All states(including near-states) are closely connected. We can say that generation 1 forms a continuum of states. Experimentally observed, we find a Lorentzian envelope of states indicating close connectivity and accessibility at high energy of the initial quasi-particle. However, our system is not ergodic (a consequence of the conservation of energy at low Temperatures).

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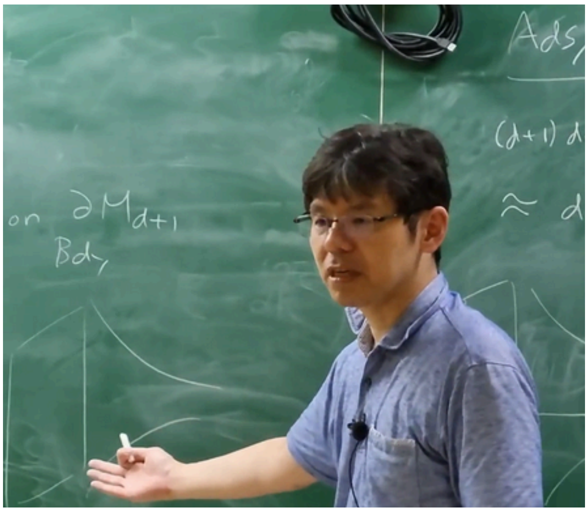
ABOUT ENSEMBLE

Ensemble is the undergraduate physics club of IISc, run by students passionate about exploring and sharing the beauty of physics beyond the classroom. The club provides a platform for learning, discussion, and collaboration through lectures, problem-solving sessions, paper presentations, and outreach activities. Some of our activities are shown below.

Participation in the IISc Open Day 2025 where we have demonstrated various experiments to the public.

Undergraduate students giving lectures on basic physics topics to their juniors.



A lecture by Prof. Tadashi Takayanagi, a pioneer in Holographic Entanglement Entropy

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Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry

Richard P Feynman